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# THE MATHEMATICS TEACHER

Volume XLIII MAY 1950 Number 5

CONTENTS


| | | Page |
|---|---|---|
| The Trigonometric Functions | John W. Coll | 181 |
| Membership Message | Mary C. Rogers | 182 |
| Pi Mu Epsilon Contests and Awards | J. R. Mayor | 183 |
| Arithmetic Section | Ben A. Sueltz | 184 |
| Using a "Calculator" to Develop Basic Understandings and Meanings in Arithmetic | Alice Rose Carr | 185 |
| Diagnosis of Disabilities in Arithmetic Essentials | Lee E. Arthur | 191 |
| Program for Meeting of NCTM in St. Louis, Missouri | | 200 |
| Cooperative Action in the Improvement of Science and Mathematics Teaching | J. R. Mayor | 202 |
| Miscellanea—Mathematical, Historical, Pedagogical | Phillips Jones | 205 |
| Institute for Teachers of Mathematics at Tufts College | | 208 |
| Group Affiliation with the National Council | H. W. Charlesworth | 209 |
| Have You Heard? | Edith Wooley | 210 |
| Aids to Teaching | Henry W. Syer and Donovan A. Johnson | 211 |
| Editorial | | 214 |
| Book Reviews | | 215 |
| News Notes | | 220 |
| Program of the NCTM, Twenty-Fourth Meeting | | 221 |


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# THE MATHEMATICS TEACHER

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*Devoted to the interests of mathematics in Elementary and Secondary Schools*

*Editor-in-Chief*—William David Reeve, Teachers College, Columbia University.
*Associate Editors*—Vera Sanford, State Teachers College, Oneonta, N.Y.
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THE MATHEMATICS TEACHER

Volume XLIII Number 5

Edited by William David Reeve

# The Trigonometric Functions

*By* JOHN W. CELL

*Professor of Mathematics, North Carolina State College*

IN THIS article we shall indicate many different methods by which the sine and cosine functions may be *defined*. (From these the other four functions may be obtained by their usual definitions in terms of the sine and cosine functions, viz., $\cot\theta=\cos\theta/\sin\theta$.) In the course of the discussion we shall consider the trigonometric functions from various points of view and we shall list properties which are not to be found in standard texts on trigonometry but which are found in advanced mathematics. We shall also indicate some general applications which are inherent in these various methods and sources for other applications.

## I. Definitions in Terms of Ordinate, Abscissa, and Distance

Of all the definitions of the trigonometric functions, the most important are the set in which we define

$$\sin\theta=\frac{\text{ordinate}}{\text{distance}}=\frac{y}{r},$$

$$\cos\theta=\frac{\text{abscissa}}{\text{distance}}=\frac{x}{r}.$$

From these two definitions and the Pythagorean theorem it is easy to prove that $\sin^2\theta+\cos^2\theta\equiv 1$. (Students need to learn how to obtain easily and quickly the other two Pythagorean identities from this identity. Too many students lack confidence in their knowledge of trigonometric identities—a defect which stems from trying to memorize all of them instead of memorizing a few from which they could obtain the others.)

The two basic reduction rules are derivable directly from these ordinate-abscissa-distance definitions: the rules for simplifying

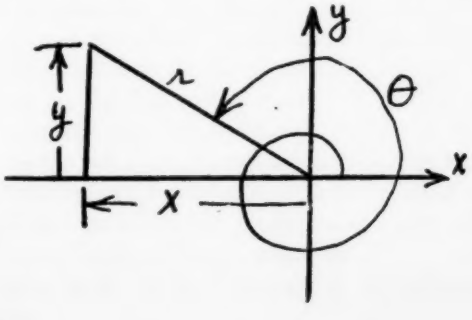
$$\text{any function (even}\cdot 90^\circ\pm\theta),$$
$$\text{any function (odd}\cdot 90^\circ\pm\theta).$$

We may utilize these same definitions to derive

$$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi,$$

$$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi.$$

The student should be afforded experience in starting with these and obtaining identities for $\tan(\theta+\phi)$, $\sin 2\theta$, $\cos 3\phi$, $\sin(A+B)+\sin(A-B)$, etc. (The typical



textbook gives too many of such identities in bold-faced type instead of encouraging the student to develop his own ability in performing algebraic manipulations with the basic identities. Notice that all of the fundamental identities that the student *must* memorize can be written in long-hand on one side of an ordinary sheet of paper.)

The student of calculus must have ready command of this "algebra of trigonometry"; for instance he should find it easy to simplify $\sqrt{4-9x^2}$ if $3x=2\sin\theta$, to change $\cos^4 7\theta$ to $[(1+\cos 14\theta)/2]^2$ to $\frac{3}{8}+\frac{1}{2}\cos 14\theta+\frac{1}{8}\cos 28\theta$, to change $\cos 7\theta \sin 3\theta$ to $\frac{1}{2}\sin 10\theta-\frac{1}{2}\sin 4\theta$ (here he ought to *start* by writing out the expansions of $\sin(7\theta+3\theta)$ and $\sin(7\theta-3\theta)$). Moreover, applications of this "algebra of trigonometry" are legion in texts in physics and engineering.

Turning from algebraic to graphical trigonometry, we may from these same definitions calculate the values of the trigonometric functions of special angles (0°, 30°, 45°, etc.). We may plot the usual graphs of $y=\sin x$ and $y=\cos x$, utilizing the angle $x$ in radian measure. We may interpret these same equations differently and use $r=\sin\theta$ and $r=\cos\theta$, and draw the corresponding circles in these polar coordinates. Applications of these circle diagrams and of the usual sine and cosine graphs are to be found in any problem involving vibration. For instance, the common home voltage wave is of the form $y=169\sin 120\pi t$ with $t$ in seconds and $y$ in volts. That this voltage is commonly designated as being about 120 volts can be explained by the following graphical method: Suppose that we interpret the equation in the form $r=169\sin\theta$ with $\theta=120\pi t$ radians. In one second the angle $\theta$ will go through $120\pi$ radians or 60 revolutions or 60 cycles. If, now, we plot the graph in polar coordinates, we obtain a circle with center on the ninety-degree line and radius 169/2. As $\theta$ takes on values from 0 to $2\pi$ radians, the radius vector traces this same circle twice. The total area traced out in one full revolution is, then, $2(169/2)^2\pi$ or $169^2\pi/2$. Now a circle, with center at the pole and radius such that the area of this new circle is the same as the preceding area, will have a radius given by $\pi r^2=169^2\ \pi/2$, whence $r=169/\sqrt{2}$, or about 120. If, instead of $r=120$, we revert to the rectangular coordinate interpretation for the same equation, we have $y=120$ volts. (Other explanations for this are to be found in texts on electricity.)

## II. Definitions in Terms of the Opposite Side, Adjacent Side, and Hypotenuse

If $\theta$ is one of the acute angles of a right triangle, then

$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}},$$

$$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}.$$

For acute angles these definitions are equivalent to the definitions in terms of the ordinate, abscissa, and distance; if the angle $\theta$ is NOT between 0° and 90° the right-triangle definitions do NOT apply.

Every text on trigonometry has applications of these definitions in terms of mensuration problems involving right and oblique triangles. The solution of oblique triangles has been taught traditionally in terms of the familiar four cases and with the laws of sines, cosines, tangents, and half-angle formulas. With the prevalent usage of high-speed computing machines, the rôle of the laws of sines and cosines has become dominant. The chief importance that we teachers can attach to the laws of tangents and half-angles is in their derivations.

## III. Definitions as Line Values

That we may define $\sin\theta$ and $\cos\theta$ as line values, or as projections of a rotating vector, is common knowledge. Moreover, such definitions are readily seen to be equivalent to the definitions in terms of ordinate, abscissa, and distance.

This concept finds application in discussions of problems involving vibration (mechanical engineering, aeronautics, electrical engineering).

We teachers find the concept of line values an aid in teaching the student the fundamental identities and the reduction formulas. It is a convenient mode of thought.

### IV. The Euler Definitions in Terms of $e^{i\theta}$

The sine and cosine functions were defined by Euler (1707–1783) as follows:

$$\sin\theta=(e^{i\theta}-e^{-i\theta})/(2i),$$

$$\cos\theta=(e^{i\theta}+e^{-i\theta})/2,$$

where $e\approx 2.718$ and $i=\sqrt{-1}$. From these definitions one can establish such identities as $\sin^2\theta+\cos^2\theta=1$, $\cos 2\theta=\cos^2\theta-\sin^2\theta$; the resulting practice in the laws of exponents is of value to the student who expects to take two years or more of college mathematics.

That these Euler definitions are equivalent to the definitions in terms of the ordinate, abscissa, and distance is not readily apparent. The equivalence is established by the fact that the sine and cosine functions, whether defined by the one method or the other, satisfy basic identities and have properties of continuity and differentiability. In this and the succeeding sections we shall step from trigonometry as such into various sections of advanced mathematics, from which vantage points we can describe other interesting properties of the trigonometric functions.

From these two Euler definitions we can show that

$$e^{i\theta}=\cos\theta+i\sin\theta.$$

We note, for example, that when $\theta=3\pi$, $e^{i3\pi}=-1$; hence one value of $\log_e(-1)$ is $i3\pi$. More generally,

$$\log_e(-1)=1n(-1)=i(2n+1)\pi$$

with $n=0, \pm 1, \pm 2$, etc. This example illustrates the fact that there are an infinitude of logarithms of any given number. For each positive real number, there is one logarithm that is a real number and an infinitude of logarithms that are of the form: the real logarithm $+i2n\pi$.

A further consequence may be obtained from the result for $\theta=\pi$: $e^{i\pi}=-1$; and from the knowledge that $e$ is a transcendental number, one can establish by methods of advanced mathematics that $\pi$ is likewise a transcendental number. From this fact one can show that the famous Greek problem of squaring the circle is unsolvable.

From these same definitions it is relatively easy to compute the sine of a complex angle as, for example, $\sin(3+4i)$—an ability needed by the advanced student in aeronautics and electrical engineering.

### V. Definitions as Solutions of Functional Equations

Of interest to teachers of trigonometry are the definitions as solutions of functional equations. We may define $\sin x$ and $\cos x$ as those solutions of the two functional equations

$$S(2x)=2\,S(x)\,C(x),$$

$$C(2x)=2[C(x)]^2-1,$$

which satisfy the conditions that $S(0)=0$ and $C(0)=1$, and which satisfy the further condition that $C(\pi/2)=0$ and $C(x)\neq 0$ for $0<x<\pi/2$.

There is no special reason for using these particular identities; we could use any two of the usual fundamental identities as the basis of this type of definition. In other words, all of the algebraic portion of trigonometry is derivable from a pair of the identities together with suitable special values for boundary requirements.

Methods of solving such functional equations are to be found in texts in advanced mathematics. Such methods basically amount to changing the problem into one involving differential equations or some other type of equation where the procedure for solution is well known.

This type of definition is used in ad-

vanced mathematics to define certain functions.

### VI. Definitions in Terms of Infinite Series

The definitions in terms of infinite power series are especially useful for computation and hence for construction of tables of trigonometric functions.

We may begin afresh and define

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{7!} - \cdots,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots,$$

with the angle $x$ in radian measure. From the rules of infinite series it can be shown that these series converge (and can be used) for *all* values of $x$. However, they are best suited for computation with small values of $x$, since the terms will then decrease rapidly in value.

One may use these two infinite series and the rules that govern the algebraic manipulation of infinite series to show that $\sin 2x = 2 \sin x \cos x$, for example, and that these definitions are equivalent to those in terms of the ordinate, abscissa, and distance.

We observe that if $x$ is quite small, then $\sin x \approx x$. A better approximation, clearly, would be $\sin x \approx x - x^3/6$, and so on. Thus, $\sin (0.1 \text{ radian}) \approx 0.1$ and $\cos (0.1) \approx 0.995$.

We may illustrate the use of these series by computing $\sin (1°)$ and $\cos (1°)$ as follows:

$$\sin (1°) = \sin (\pi/180 \text{ rad.})$$

$$= (\pi/180) - \frac{(\pi/180)^3}{3!} + \frac{(\pi/180)^5}{5!} - \cdots$$

$$= 0.0174533 - 0.00000088 + \cdots$$

$$\approx 0.017453,$$

$$\cos (1°) = 1 - (\pi/180)^2/2 + (\pi/180)^4/24 - \cdots \approx 0.999848.$$

From these results and the knowledge of the values of the sine and cosine of the special angles we may compute, for example, the values of $\sin 31°$ and $\cos 44°$.

### VII. Definitions as Solutions of Differential Equations

Teachers of trigonometry will find in this and in the following sections facts which, while of little value in their teaching, will nevertheless be of interest in giving a rounded picture of trigonometry.

We may define $\sin x$ and $\cos x$ as those solutions of the differential equation

$$\frac{d^2y}{dx^2} + y = 0,$$

which satisfy certain initial or boundary conditions. For $\sin x$ we could require that $y = \sin x = 0$ when $x = 0$, and that $dy/dx = 1$ when $x = 0$. For $y = \cos x$ we could require that $y = 1$ when $x = 0$, and $dy/dx = 0$ when $x = 0$.

We may solve this differential equation for the two solutions that satisfy the two given sets of numerical conditions. If we solve for the two solutions in infinite power-series form, we will obtain the infinite series already used to define these two functions in section VI. Alternately, we may assume $y = ae^{mx}$ to be a solution by which we are led to the solution of the algebraic equation $m^2 + 1 = 0$. If we then take the general solution to be $y = ae^{ix} + be^{-ix}$ and apply the given numerical conditions, we are led to the Euler definitions in terms of the exponential function as given in section IV.

As another method of using differential equations, we could define $y = \sin x$ and $z = \cos x$ as those solutions of the simultaneous differential equations

$$dy/dx = z \text{ and } dz/dx = -y$$

that satisfy the conditions: $y = 0$ and $z = 1$ when $x = 0$.

Differential equations are used as the genesis of many functions of advanced mathematics. For example, the Bessel functions are often defined as solutions of

a particular differential equation from which the properties of the Bessel functions are derived.

### VIII. Definitions as Definite Integrals

The multifarious facets of trigonometric functions are further exemplified in integral calculus. We define

$$\text{arc sin } y = \int_0^y \frac{dw}{\sqrt{1-w^2}},$$

$$\text{arc tan } y = \int_0^y \frac{dw}{1+w^2}.$$

Since these definitions define these two inverse trigonometric functions only for the range of principal values, we need to adjoin to these definitions certain requirements of continuity or of periodicity in order to obtain the equivalent of the general definitions already given.

Many functions are defined, as regards their genesis, as definite integrals. For example, the gamma function satisfies

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx, \quad n > 0.$$

On the other hand, if $n$ is a positive integer, $\Gamma(n+1) = n!$. Articles which treat new functions defined as definite integrals are numerous in the advanced journals of science, engineering, and mathematics.

### IX. Definitions as Special Series

If we integrate by parts the two definite integrals treated in the preceding section, we are led to the following series:

$$\text{arc sin } y = \frac{y}{(1-y^2)^{1/2}} - \frac{1}{3}\,\frac{y^3}{(1-y^2)^{3/2}} - \frac{1}{5}\,\frac{y^5}{(1-y^2)^{5/2}} - \cdots,$$

$$\text{arc tan } y = \frac{y}{1+y^2} + \frac{2}{3}\,\frac{y^3}{(1+y^2)^2} + \frac{2}{3}\,\frac{4}{5}\,\frac{y^5}{(1+y^2)^3} + \cdots.$$

The first series converges (is usable) for $-0.707 < y < 0.707$, and the second series converges for all real values of $y$. These two series may, of course, be used as the genesis of the algebraic portion of trigonometry.

The prime reason for mentioning these peculiar series is that when certain other definite integrals for functions of advanced mathematics are integrated in the same manner, the resulting series are divergent but are also in a special class of series which are known as asymptotic series and may be used, subject to certain rules, for computation purposes.

### X. Definitions as Infinite Products

We may define sin $x$ and cos $x$ anew by the following infinite products:

$$\sin x = x \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{\pi^2 n^2}\right) = x\left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{4\pi^2}\right)\cdots,$$

$$\cos x = \prod_{n=1}^{\infty} \left(1 - \frac{4x^2}{\pi^2(2n-1)^2}\right).$$

These infinite products portray the sine and cosine functions in factored form. These are equivalent definitions for these two trigonometric functions whose proof involves advanced mathematics.

### XI. Definitions as Continued Fractions

We could, if we wished, start with the following as the definition for tan $x$:

$$\tan x = \cfrac{x}{1 - \cfrac{x^2}{3 - \cfrac{x^2}{5 - \cfrac{x^2}{7 - \ddots}}}}$$

This definition is valid for all values of $x$ except, of course, those values of $x$ for which tan $x$ is not defined.

If we compute the successive values of this fraction for $x = 1$, we obtain the following sequence of values: $1/1 = 1$,

$1/(1-\frac{1}{3})=1.5$, 1.55556, 1.55738, etc. The value of tan (1 radian) is 1.5574 correct to four decimals.

These eleven methods by which we have defined the trigonometric functions are not all of the methods possible. However, these are sufficient to permit us to say that there are many different ways of defining the trigonometric functions. The definitions given in the latter part of this paper could properly be made the basis of an unusual and stimulating summer course in advanced calculus for teachers of high school and elementary college mathematics.

## Membership Message

*By* MARY C. ROGERS

Your National Council is happy to submit this second listing of Membership Honor Schools for the year 1949–1950. We are glad you so heartily endorse this plan for special membership recognition for it indicates your keen interest in the work of the Council. It bespeaks your sincere support of Council plans and activities, and your willingness to assist in achieving the many services for Mathematics Education which we are increasingly setting into motion.

Quite a number of you have written us, reporting your membership status even though your school does not now qualify as an Honor School. Thank you for this expression of interest. We shall anticipate including you on future Honor Lists in later issues of THE MATHEMATICS TEACHER.

Definite plans as to frequency of Membership Reports for next year have not yet been formulated, but you will hear from us again next school term. In the meantime, please accept our sincere wishes for a very pleasant vacation. We hope we may see a goodly number of you at the Summer Convention at Madison, Wisconsin in August.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS MEMBERSHIP RECORD

*100% Schools as of March 15, 1950*

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| 4. | Washington, D. C. | Calvin Coolidge High School |
| 5. | Washington, D. C. | Gordon Junior High School |
| 6. | Washington, D. C. | Langley Junior High School |
| 7. | Washington, D. C. | McKinley High School |
| 8. | Washington, D. C. | Roosevelt High School |
| 9. | Washington, D. C. | Stuart Junior High School |
| 10. | Washington, D. C. | Taft Junior High School |
| 11. | Washington, D. C. | Veterans High School Center |
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| 13. | Maywood, Illinois | Proviso Township High School |
| 14. | Riverside, Illinois | Riverside Brookfield High School |
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| 17. | Passaic, New Jersey | Passaic Collegiate School |
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| 19. | Nashville, Tennessee | Central High School |
| 20. | San Marcos, Texas | San Marcos Junior-Senior High School |
| 21. | Madison, Wisconsin | Wisconsin High School of the University of Wisconsin |

*"All but One" Schools as of March 15, 1950*

| | | |
|---|---|---|
| 1. | Rison, Arkansas | Rison Public High School |
| 2. | Lansdowne, Pennsylvania | Lansdowne High School |
| 3. | Straw Plains, Tennessee | Carter High School |

# Pi Mu Epsilon Contests and Awards

*By* J. R. MAYOR
*The University of Wisconsin*

QUITE a number of the chapters of Pi Mu Epsilon, an honorary mathematics fraternity for university undergraduate and graduate students, sponsor competitive activities for mathematics students. A good many readers of THE MATHEMATICS TEACHER no doubt have won Pi Mu Epsilon awards during their college days and a great many more of their students should be future winners of these contests. While most of these activities are sponsored for college students the nature of the contests and the kinds of recognition given should be suggestive for those interested in contests for high school students.

An understanding of the purposes of Pi Mu Epsilon may be obtained from a statement by C. C. MacDuffee, Director-General, in the first issue of the new *Pi Mu Epsilon Journal*, November, 1949. Professor MacDuffee states that he is "convinced that Pi Mu Epsilon has an important mission in American university life in pointing out to competent students the beauty and utility of mathematics and its related sciences, and in insisting that mathematics be accorded its rightful place in the curriculum," and he adds, "Surely we can defend the thesis that mathematics deserves an important place in the education of every competent man and woman."

The information contained in this section was reported in response to a request sent to 46 chapters of Pi Mu Epsilon. Reference was made in the author's October report to the contests of several Pi Mu Epsilon chapters. There is also a brief article on "Medals, Prizes and Scholarships" in the first issue of the *Pi Mu Epsilon Journal*, pages 34–35.

The St. Lawrence University, Canton, New York, chapter reports an annual contest for eight high schools in its vicinity. Each school sends three participants for three-hour tests in algebra, geometry, and trigonometry. The prizes are medals and certificates for individuals and team winners and a silver cup which is retained by the school which wins the cup three times.

The University of Montana chapter offers three cash prizes ($25, $15, $10) to incoming freshmen from Montana high schools who rank highest on an entrance examination. Prize money comes from funds contributed by alumni of the chapter. The chapter also annually presents books to the most outstanding student in mathematics and the most outstanding student in the field of physics.

Prizes based on freshman work are awarded by the chapters at The Ohio State University, Oklahoma A & M College, and The University of Nebraska. At the Oklahoma college the record in all subjects is also taken into consideration. Students who have completed a year's work in the calculus are eligible for mathematics contests at the University of California at Los Angeles, The University of Missouri, University of Oklahoma, and The University of Nebraska.

The University of Illinois chapter grants an award based on the general average of the student and accomplishment in mathematics for the four years of university work. Saint Louis University Pi Mu Epsilon members sponsor an essay contest on the life and works of a famous mathematician. Freshmen and sophomores write on the life of the mathematician while all undergraduates are eligible to write an essay on the works of the mathematician chosen.

The University of Kansas and the University of California at Los Angeles have awards for those who solve (and have the solutions accepted for publica-

tion) problems published in *The American Mathematical Monthly*.

In addition to cash prizes ranging from $25 to $1, cash awards for purchase of books on mathematics and those awards mentioned earlier in this report, other kinds of awards given to winners of competitions in mathematics sponsored by Pi Mu Epsilon chapters include:

Silver and gold keys of appropriate design
Books
Name engraved on plaque
Signed scroll
Name read at university convocation
Statement written on official transcript of student
Publication of essay in chapter news letter.

The secretary of The University of Missouri chapter writes that "Our purpose is not to discover genius, but to reward accomplishment and to encourage interest in mathematics." At Nebraska it is intended that "understanding and originality more than mechanical proficiency" be tested.

---

# Arithmetic Section

*Conducted by* BEN A. SUELTZ
*Cortland, N. Y.*

IN THIS ISSUE:

Miss Carr describes how to use a type of abacus to develop meanings and understandings in primary arithmetic. Note how her "calculator" is much more useful than an ordinary frame abacus. Another important feature is that pupils can see and handle the counters: they are genuine participants in the initial stages of learning. This calculator is useful beyond simple counting and number combinations, in fact it has many potential uses which the discerning teacher will recognize. Miss Carr speaks of "borrowing" in subtraction when the actual operation is one of "changing" or "decomposition." Has the time come when we should, in the interests of understanding, drop the word "borrow" as conventionally used in subtraction? What other materials and devices are useful in enhancing the teaching of arithmetic?

In another article Mr. Lee E. Arthur discusses procedures used in diagnosis of difficulties in arithmetic. Perhaps the most significant part of his study is the recognition that pupils are weak in arithmetic, that these weaknesses should be located, and that the teachers in the high school should do something about the situation. We would have much less complaint about the mathematical illiteracy of our high school graduates if more schools recognized the functional importance of arithmetic and assumed the responsibility therefor at the high school level. Is it fair to deny a pupil the opportunity to learn arithmetic in the junior or senior high school? You may not agree with the scope of testing used by Mr. Arthur. He may have omitted certain functional aspects such as concepts, information, principles, ideas, and judgments which frequently are used in non-computational situations.

# Using a "Calculator" to Develop Basic Understandings and Meanings in Arithmetic

*By* Alice Rose Carr
*Ball State Teachers College, Muncie, Indiana*

In the past few years, many teachers have turned more and more to the use of concrete materials to develop with children an understanding of the number system and the processes involving number relationships. Too often the teacher has put the emphasis upon abstract manipulation of number symbols rather than upon the understanding of basic concepts which should precede the use of symbols. It is important that the child understand "why" he does "what" he does.

A simple device which has been helpful in teaching many topics in arithmetic is an adaptation or modification of the familiar abacus. It consists of several spindles mounted vertically on a wooden tray in such a way that counters similar to checkers may be put on and taken off. Counters on the first spindle at the right represent *ones;* those on the second spindle *tens;* on the third, *hundreds.* Counters are of different colors to indicate their value. The number of spindles used on the calculator depends upon the size of the numbers being used by the class.

As children count, they put the counters one by one on the first spindle until they reach ten. All ten counters are then removed and exchanged for one of a different color which is placed on the second spindle to represent one group of ten. Counting then continues until another ten *ones* are on the first spindle and are again exchanged for one *ten.* The hundreds' spindle comes into use as soon as ten *tens* have been counted.

In its representation of the numbers, the calculator differs very little from the abacus, the only difference being that the ten *ones* are actually removed from the spindle when exchanged for a *ten.* However, as we discuss the use of the calculator in the addition and subtraction processes, it will be seen that there is a distinct advantage in having the spindles long enough to accommodate more than ten counters.

In addition, "carrying" is very easily explained by means of this calculator. If we are considering exercises in which only two addends are involved, we need the spindles long enough to hold eighteen counters, since the largest possible combination is 9+9. Let us consider the problem $\begin{array}{r} 26. \\ +38 \\ \hline \end{array}$ We first lay out the counters (two *tens* and six *ones;* three *tens* and eight *ones*) used to represent the numbers to be added. When we place the eight *ones* and the six *ones* on the first spindle, we find that there are fourteen *ones.* Ten of these are removed, exchanged for a *ten* which is then placed on the second spindle with the two *tens* and three *tens.* Our answer is then read from the calculator, 64—six *tens* and four *ones.* The process is very simple and brings out clearly the meanings which we wish to teach children.

In more difficult problems, "carrying" more than once is simply an extension of a process already understood. Calculations involving the use of zero are made meaningful by the absence of counters on one or more spindles.

In teaching "borrowing" in subtraction, we reverse the process used in "carrying" in addition. Let us consider a more difficult example: $\begin{array}{r} 7004. \\ -2685 \\ \hline \end{array}$ The pupil sets up the minuend, 7004, on the calculator by placing seven counters on the fourth spindle and four on the first. Zeros are indicated by empty spindles. Let us sup-

pose that he has chosen red counters for *thousands*, green for *hundreds*, orange for *tens*, and blue for *ones*. As he begins to subtract, he is confronted immediately with the problem of taking off five *ones* when there are only four there. Since there are no *tens* and no *hundreds*, he must: (a) remove one of the seven red counters (*thousands*) and exchange it for ten green counters (*hundreds*) which he places on the *hundreds'* spindle; (b) remove one of these and exchange it for ten orange counters (*tens*) which he places on the *tens'* spindle; (c) remove one of these and exchange it for ten blue counters (*ones*) which he places with the four already on the *ones'* spindle. The calculator now shows six *thousands*, nine *hundreds*, nine *tens*, fourteen *ones*. He finishes the example by taking off the counters as indicated in the subtrahend 2685 and has his answer left to read on the calculator, 4319.

The use of this calculator is by no means limited to the primary grades. In the junior or senior high school it helps to make meaningful many of the computational processes which pupils may have been performing without an understanding of their basic significance. It is possible to show simple problems in multiplication and long division on the calculator. In the upper elementary grades, it can be used in the reading of large numbers. For this purpose, one should have at least seven spindles so as to read numbers in the millions. If one wishes to show decimals on the calculator, all that is necessary is to lay some article such as a pencil between two spindles to indicate the decimal point.

The chief value of an instrument such as this calculator lies in the fact that properly used it can lead children to real insight into the meaning of our number system. The children themselves enjoy using it and beg to be the one to put on the counters or take them off. The actual participation on their part helps to hold their interest and fixes in their minds the significance of what otherwise might be mere rote memory or abstract juggling of symbols.

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## 1950 Intersession and Summer Session, School of Education, New York University, Courses for Teachers of Mathematics

The School of Education, New York University, will offer the following courses in the teaching of mathematics in the intersession of 1950, June 6–June 30:

By Professor John J. Kinsella:

1. Preparation and use of resource materials in teaching mathematics
2. The use of teaching aids in mathematics education

For the summer session, July 5–August 11, the following courses will be offered:

By Dr. Aaron Bakst:

1. Teaching of mathematics in the senior high school

By Professor John J. Kinsella:

1. Current trends in mathematics curriculums and teaching procedure
2. The use of tests and other evaluation procedures in mathematics education

By Dr. Amanda Loughren:

1. Teaching and supervision of arithmetic

By Mr. Harry S. Ruderman:

1. Teaching of junior high school mathematics

# Diagnosis of Disabilities in Arithmetic Essentials

*By* Lee E. Arthur

*Riverside-Brookfield High School, Riverside, Illinois*

## I. Introduction

For many years teachers of mathematics have been criticized for the inability of high school graduates to apply the mathematics which they were supposed to have learned in the elementary and high schools. Critics varied from the shop owner who complained that his help could not add and subtract numbers to the engineering professor who claimed that his students did not have sufficient understanding of basic principles to carry on advanced study in his courses.

To this chorus of criticism was added the voices of those responsible for training the armed forces during the late war. The report, "Essential Mathematics for Minimum Army Needs"[1] which was published in 1943 continues to be valuable for teachers of mathematics. In this report the committee (henceforth called Committee on Essentials) claimed that enrollment in advanced courses does not guarantee proficiency in the use of minimum essentials, nor does accuracy in computation assure facility in the interpretation of actual problem situations. They further declared that reteaching or reorientation in instruction, rather than drill on forgotten operations, is necessary if deficiencies are to be corrected. They stated that those who enter the services or civilian lines of work must be trained in the ability to meet quantitative situations effectively, confidently, and sensibly. Specifically, they stated that pupils must be taught to, a) deal with these situations by approximations and estimations, b) identify the quantitative aspects of a situation, c) recognize and use the symbolism of mathematics, d) tell when and how symbolism and procedures are to be used, and e) compute accurately and quickly.

The committee prepared a list of forty-nine items which they considered to be the minimum essentials for the basic training program. Of these about thirty could be classified as belonging to the field of arithmetic and the other nineteen to algebra and geometry. Each item was discussed as to degree of difficulty likely to be encountered. The report continued with an extensive discussion of how the basic principles might be taught.

## II. Purpose of This Study

A study of the report of the Committee on Essentials prompted the writer of this paper to investigate the situation in his own school in regard to the disabilities of pupils in the essential mathematics outlined by the committee. The cooperation of the other teachers of the mathematics department was enlisted. They proceeded to construct an instrument which they believed would determine in which of the essentials their pupils were deficient. Since the items belonging to the fields of algebra and geometry would receive particular attention in the regular courses in these subjects, it was decided to limit the study to those items belonging to the field of arithmetic. Since all freshmen entering the high school in which the study was conducted would have completed the study of arithmetic as a subject and presumably would have had opportunity to acquire skill in the arithmetic essentials listed by the committee, it was decided to construct an arithmetic test which would determine their proficiency on each of the arithmetic items listed. The results of such a test would indicate which of the essentials needed further attention in high school. Ample opportunity is provided for

[1] "Essential Mathematics for Minimum Army Needs." The Mathematics Teacher, October, 1943.

the correction of disabilities located since nearly 100% of the freshmen study either algebra or general mathematics. The responsibility for making these corrections whatever their nature, rests with the teachers of high school mathematics.

### III. The Test

Each of the arithmetic essentials listed by the Committee as being the minimum for training in the armed services was carefully studied, and materials suitable for testing each of them were collected. In the preliminary draft three questions were selected to test each essential. The questions were arranged in three parts, one question in each part on each of the items to be tested. The pupils were to compute the answers and record them on a separate answer sheet. Test items were so selected that understanding of basic principles as well as computational skill was required. The original draft included ninety items.

This preliminary test was given to 125 freshmen with whom it was found to be statistically reliable. However, it contained several ambiguities and weaknesses and it was too long for one class period. The revised test which could be given in one hour included the following items listed by the Committee on Essentials.

1. Reading and writing whole numbers
2. Reading fractions
3. Reading decimals
4. Reading per cents
5. Understanding percentage
6. Addition of whole numbers
7. Subtraction of whole numbers
8. Multiplication of whole numbers
9. Division of whole numbers
10. Addition of fractions
11. Subtraction of fractions
12. Multiplication of fractions
13. Division of fractions
14. Addition of decimals
15. Subtraction of decimals
16. Multiplication of decimals
17. Division of decimals
18. Finding parts of a fraction
19. Finding parts of a decimal
20. Finding percents of numbers
21. Finding what fractional part one number is of another
22. Finding what part one decimal is of another
23. Finding what per cent one number is of another
24. Finding the whole when a fractional part is given
25. Finding the whole when a decimal part is given
26. Finding the whole when a percentage is given
27. Finding the ratio of two numbers
28. Finding one term of a proportion
29. Finding the square root of a number
30. Finding values from a scale

In its revised form the test consisted of two parts, one question in each part to test each of the thirty essentials listed above. The items of Part I were selected to test the pupil's ability to compute. The operation to be performed was indicated by use of a key word such as 'sum,' 'multiplying,' or 'quotient.' The thirty items of Part II consisted of verbal problems which the pupil had to interpret and in which he had to determine for himself the operation to be performed. Interpretation problems were used because the Committee on Essentials had stated emphatically that inductees were not able to determine what process to use in solving problems.

The test was made up of multiple-choice questions. Each question was followed by five possible choices for the answer, four of which were numerical values and the fifth 'no answer given.' Some of the questions did not have the correct answer given as one of the choices. Pupils are less likely to guess if they know that the correct answer may be missing. The numerical values given as choices were carefully selected from answers given by pupils on the preliminary form. Thus a study of the answer selected by the pupil would indicate what mistake he may have made in working the problem.

The following questions taken from the test illustrate the type of problems used and the types of values used as the possible answers. Number 14 tests the pupil's ability to add decimal fractions, while number 44 tests his ability to interpret a verbal statement which calls for the addition of decimals.

14. The sum of .025, 0.25, 2.5, and 25 is: 1) 27.775, 2) .100, 3) 28.00, 4) 25.525, 5) answer not given.
44. The perimeter of a plot of ground whose dimensions are 75.95 ft., 53.06 ft., 120.70 ft., and 39.48 ft. is: 1) 72.3 ft., 2) 289.2 ft., 3) 299.2 ft., 4) 144.6 ft., 5) answer not given.

The pupil was to select the correct answer to the problem from the choices given and record his choice by underscoring the number on the answer sheet corresponding to the number of the choice selected for his answer.

The test was set up in "multiple-choice" form for ease of scoring and because this form is more satisfactory for determining abilities and deficiencies than is the demonstrative type of test. A recent study corroborates the validity of this type of test.[2]

The revised form of the test was administered to 194 freshmen during the first week of school in September, 1945. Analyses of the results showed that the test was almost ready for final form. A few minor revisions due to ambiguous questions were necessary. Some further revisions were made so that the test could be scored on the International Business Machines test scoring machine which had been installed in the school. These revisions were made and the test administered to 210 freshmen in September, 1946 and to 199 in September, 1947.

In its present form the answer sheets for an entire class may be scored on the IBM machine in a few minutes, or by hand in a very short time by use of a perforated stencil placed over the answer sheet. Check marks through the perforations give a permanent record of items missed by each pupil, to be analyzed and discussed with him when convenient.

## IV. Analysis of Results

The test was constructed for the express purpose of determining disabilities of pupils on those essentials listed by the Committee on Essentials. Therefore, it was not deemed necessary to make extensive statistical analyses of the results. Rather, analyses of results called for a study of the incorrect answers of individual pupils on each of the items tested. Such a study would indicate not only the operation missed but the types of errors made and the type of remedial work necessary.

However, even a diagnostic test of this type should possess some degree of reliability. Computations of the reliability coefficients for each part of the test and for the test as a whole were made. The results are shown in Table I. Reliability coefficients were computed by use of the rational equivalence formula $1-(\Sigma pq/\sigma^2)$. $p$ is the ratio of the correct responses to the number of answers given for each item, while $q$ is the ratio of the number of incorrect responses to the number of answers for each item.

Table I

*Reliability Coefficients*

| | |
|---|---|
| Part I | .75 |
| Part II | .81 |
| Total | .88 |

These coefficients are high enough to indicate that the test is sufficiently reliable for measuring the abilities tested. The reliability coefficients could be raised by lengthening the test. However, this would not be desirable because of the time limit placed on it.

The scores of the test were correlated with those made on the arithmetic test of the Stanford Achievement Test. While these coefficients were not particularly high, about $.70 \pm .02$, it was believed that this fact was not particularly significant since the Stanford Test is used for determining pupils' achievements in arithmetic while the arithmetic essentials test is used for diagnosing specific difficulties.

Of greater significance for this study is a comparison of scores made on the two parts of the arithmetic test. Such a com-

[2] "Comparison of Problem Types in the Comprehensive Mathematics Test." Lynett B. Plumlee, *The College Board Review*, Fall, 1947.

parison is made in Table II, as follows:

TABLE II

*High, Low, and Median Scores*

| | Part I | Part II | Total |
|---|---|---|---|
| High Score | 30 | 29 | 57 |
| Low Score | 6 | 2 | 8 |
| Median Score | 19.3 | 12.8 | 32.2 |
| Possible Score | 30 | 30 | 60 |

Table II shows that pupils made much lower scores on Part II since the median score on Part II was 12.8 and on Part I, 19.3. An examination of individual scores showed that in nearly every case pupils made lower scores on Part II than on Part I. These results were to be expected since Part II consisted entirely of verbal problems in which the pupil had to determine the operation to be performed. Teachers of mathematics are well acquainted with the fact that for most pupils interpretation of verbal problems is one of the most difficult mathematical abilities to master. Pupils with low mental ability were, of course, particularly inept at interpretation.

The results shown in Table II indicate that much remedial teaching is necessary before many of the pupils could be expected to perform satisfactorily the operations called for by the essentials tested. What form the remedial teaching was to take and what essentials were to be stressed would be determined from the analysis of individual errors made.

The correlation coefficient of scores on Part I compared to the computation scores on the Stanford was $.72 \pm .02$. Scores on Part II correlated with the scores on the reasoning part of the Stanford gave a coefficient of $.71 \pm .02$. In general, this would indicate that those pupils who made a high score on the Stanford Test also made a high score on the arithmetic test, and those who made a low score on one made a low score on the other.

It was found that of the sixty test items only twenty-one were answered correctly by 75 per cent or more of the pupils, fourteen items on Part I, or the computation part of the test, and seven items on Part II, the interpretation part. However, it must not be assumed that any operation which has been worked incorrectly by only a few pupils has been performed satisfactorily and needs no further attention by the group. Any problem worked incorrectly by any pupil indicates a disability for him, and all such individual disabilities should be corrected regardless of how seldom they occurred in the group. Any item answered incorrectly by a small number of pupils may be retaught individually or to small groups, while items missed by a large number can be retaught more economically as a class project.

A complete analysis of all disabilities found from a study of the test results is entirely too long for this paper. However, certain patterns of disabilities do become evident and are discussed below.

1. As expected, many more errors were made in Part II than in Part I. Many of the pupils were unable to interpret many of the statements and so could not determine the operations to be performed. Unless some key word was contained in the problem or the problem was stated after some familiar pattern many errors in interpretation were made. For example, the problem calling for the addition of three fractions with unlike denominators was computed correctly by 81 per cent when stated as a sum, but a problem involving the same operations with fractions of equal difficulty when stated as "the distance around a triangle" was solved correctly by only 24 per cent.

2. Problems in computing the whole when a part was given, or in finding what part one number is of another were particularly troublesome. This was true whether the problem was one of computation or one of interpretation. The following items accounted for more errors than any of the others: a) finding what per cent one number is of another, b) finding the whole when a fractional part is given, c) finding the whole when a decimal part is given, d) finding the whole when a per-

centage is given. Fewer than one-half the pupils solved these items correctly on Part II and only slightly more than one-half on Part I. However, these operations are not met as frequently as most of the other essentials and so may have been forgotten because of lack of practice. These types are recognized as being difficult operations and usually cause much trouble. Such problems call for considerable reteaching at the high school level.

3. The results of the test, in general, corroborate the fact that pupils have much greater difficulty in performing the fundamental operations with decimal fractions and with proper fractions than with integers. Pupils often performed correctly on this test those operations dealing with whole numbers but worked incorrectly many problems which seemed to be of no greater difficulty, except that they involved decimals or fractions. This was found to be true when the operation to be performed was stated, as in Part I, as well as when the pupil had to determine the operation, as in Part II. Pupils may have understood the use of decimals and fractions at one time, but when they entered high school they could not use them with as great facility as they could whole numbers. This may be due in part to the fact that they encountered decimals and fractions much less frequently. This test pointed out just what operations with decimals and fractions were missed and just what the mistakes were, and indicated that these operations need considerable reteaching at the high school level.

4. Analyses of individual errors showed that a large variety of types of errors was made. Such errors as the following were very common: a) performing the wrong operation, such as adding when the problem called for multiplication, b) placing the decimal point in the wrong position when multiplying or dividing, c) making wrong number combinations, d) adding denominators when adding fractions, e) failing to invert the divisor when dividing fractions, and f) failing completely to give correct interpretations for verbal problems. A long list of such types of errors could be compiled but the above will serve to show the nature of mistakes made.

## V. Conclusions

1. A study of results made by pupils on this test tend to verify the contentions of the critics previously mentioned, that many high school pupils do not have adequate understanding of nor ability to work many of the problems which are considered essential.

2. Many of these disabilities are present when pupils enter secondary school, and if high school graduates are to have an adequate understanding of the basic arithmetic needed for quantitative manipulation and analysis teachers of secondary school mathematics must stress these essentials throughout the high school courses, not only as review but as material which must be retaught.

3. Particular emphasis must be placed on the interpretation of problem situations. Pupils are weakest in solving problems of this type. Unless problems are stated in familiar terminology pupils often become confused and are not able to determine the operations to be performed.

4. Remedial teaching of these essentials should be carried on in all high school mathematics classes. It is reasonable so suppose that these basic concepts have been taught in the elementary school and many pupils either have failed to understand them in the first place or else have forgotten them. It is as reasonable to suppose that these concepts will not be entirely retained after the first reteaching in the high school. Since they must form part of the educational equipment of most graduates they must be rechecked and retaught at frequent intervals.

5. A study of the results of such a diagnostic test as the one discussed in this paper should clearly indicate those essentials which need reteaching. Essentials missed by a few pupils may be retaught individually while those missed by many

may be retaught more economically as a class project.

6. Administration of the test, or an equivalent form of the test, at the end of the year after the necessary reteaching had taken place would indicate how successful such reteaching had been and to what extent it needed to be repeated.

7. The results of such a test are valuable only insofar as the individual errors are analyzed and instruction is instituted to correct the wrong thinking which produced the errors. The authors of this test are satisfied that it is entirely feasible for high school teachers to construct short, objective type, easily scored tests which they may use to determine what disabilities pupils have in basic arithmetic essentials. Such tests, if carefully constructed and results analyzed, would make it possible for teachers to determine easily what arithmetic must be retaught if their pupils are to have a functional grasp of arithmetic when they graduate from high school.

## Program for the Meeting of the National Council of Teachers of Mathematics Sheraton Hotel, St. Louis, Missouri, Monday, July 3, 1950

8:00 to 10:15 A.M., Registration

10:15 to 11:45 A.M., Morning Session: Meetings of Sections A and B

*Section A. Arithmetic in Grades One to Eight*, Crystal Room

Presiding: John R. Mayor, University of Wisconsin, Madison, Wisconsin

WHAT CONSTITUTES GOOD TEACHING IN ARITHMETIC?
C. N. Mills, Illinois State Normal University, Normal, Illinois

THE MEANING OF MEANING
H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa

TWO-FIGURE DIVISORS
Jesse Osborn, Harris Teachers College, St. Louis, Missouri

*Section B. General Mathematics in Grades Nine to Twelve*, El Cortez Lounge

Presiding: Kenneth E. Brown, University of Tennessee, Knoxville, Tennessee

MATHEMATICS THAT IS FUNCTIONAL IN 1950
H. C. Christofferson, Miami University, Oxford, Ohio

OBJECTIVES OF MATHEMATICS COURSES IN GRADES NINE AND TEN
Lenore John, Laboratory School, University of Chicago, Chicago, Illinois

FACTS AND FUN WITH FIGURES
C. B. Read, University of Wichita, Wichita, Kansas

12:30 P.M., *Discussion Luncheon*, Caprice Ballroom. Each table will have a discussion topic and a discussion leader. Reservations for each table will be limited to ten people. Tickets for the luncheon should be purchased and table reservations made at the registration desk at the time of registration.

2:00 to 3:30 P.M., Afternoon Session: Meetings of Sections C and D

*Section C. Algebra and Geometry in Grades Nine to Twelve*, Crystal Room

Presiding: G. H. Jamison, State Teachers College, Kirksville, Missouri

PREREQUISITE TO MEANING
Gertrude Hendrix, Eastern Illinois State College, Charleston, Illinois

THE ROLE OF ALGEBRA IN GENERAL EDUCATION
Harold P. Fawcett, Ohio State University, Columbus, Ohio

(Third speaker to be announced later)

*Section D. Arithmetic and General Mathematics in the Upper Grades and the Junior High School*, El Cortez Lounge

Presiding: Philip Peak, University School, Indiana University, Bloomington, Indiana

HOW CAN WE MAKE MATHEMATICS MEANINGFUL TO JUNIOR HIGH SCHOOL STUDENTS?
Mildred Nielsen, Cossitt School, LaGrange, Illinois

MOTIVATING ADOLESCENTS IN ARITHMETIC
Olive Leskow, Tolleston School, Gary, Indiana

SOME USES OF GRAPHICAL METHODS IN THE MEDICAL SCIENCES
Gaylord C. Montgomery, John Burroughs School, Clayton, Missouri

*Room Reservations:* All applications for room reservations should be sent directly to the Sheraton Hotel, St. Louis 8, Missouri.

*Luncheon Reservations:* There will be accommodations for about 150 people at the discussion luncheon. Applications for advance reservations for the luncheon should be sent to Dr. Ella Marth, Harris Teachers College, St. Louis 12, Missouri. Tickets for the luncheon may be purchased at the registration desk at the time of registration.

*Registration:* Registration for the meeting should be made as early as possible on Monday morning. The registration desk will be open at 8:00 A.M.

*Chairman:* The chairman of the committee on local arrangements is Miss Ella Marth, Harris Teachers College, St. Louis, Missouri. The program chairman for this meeting is Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan.

# Cooperative Action in the Improvement of Science and Mathematics Teaching

J. R. Mayor
*The University of Wisconsin*

In order to take advantage of the opportunity to discuss common problems with science teachers The National Council of Teachers of Mathematics held a one and one-half day meeting in New York at the time of the Christmas meetings of the American Association for the Advancement of Science. The suggestion for a joint session with the National Science Teaching Societies came originally from the Cooperative Committee on Science and Mathematics Teaching, a committee of the American Association for the Advancement of Science. The Cooperative Committee includes representatives of sixteen professional organizations and the United States Office of Education. E. H. C. Hildebrandt was the first representative of The National Council of Teachers of Mathematics on the Committee and the author is the present representative. Raleigh Schorling is the Committee representative of the Mathematical Association of America.

Other groups represented on the Committee are:

American Association of Physics Teachers
American Astronomical Society
American Chemical Society
American Institute of Physics
American Nature Study Society
American Society of Zoologists
Botanical Society of America
Central Association of Science and Mathematics Teachers
Division of Chemistry Education of the American Chemical Society
Geological Society of America
National Association of Biology Teachers
National Association for Research in Science Teaching
National Science Teachers Association
Section Q, Education, A.A.A.S.

The projects and recommendations of the Cooperative Committee have been reported in a number of periodicals. Those reports of most interest for teachers of mathematics are listed in the bibliography at the end of this article. Mathematics teachers may also have heard some of the Committee sponsored forums on the radio or at meetings of professional organizations. The most widely read and the most important report of the Committee has been the Committee's contribution to *Science and Public Policy*, A Report to the President by The President's Scientific Research Board.

The Cooperative Committee has been concerned primarily with problems related to (1) science and mathematics in general education at all levels, elementary, secondary, and junior college; (2) the early identification of talented youth and its proper encouragement at pre-college levels; (3) the training of science and mathematics teachers; (4) adult education in science and mathematics.

The importance of the Committee lies in its sponsorship by the great national organization of active research scientists, in the nature of its membership as representative of the various scientific professional groups, and by the opportunity it provides for cooperative planning and action of science and mathematics teachers. The Committee has pointed out to university science departments that it is their first duty to seek the cooperation of departments of education, and no agency is in a better position to work effectively with both science and education people. The benefits which might come from drawing the attention of college and university science departments and scientists in government and industry to the problems of the elementary and secondary schools could be far-reaching. Likewise great benefits can come from keeping the

importance of science and mathematics before those in education and school administration.

This year a representative of the United States Office of Education has been added to the Committee and the Committee's recent New York meeting was largely devoted to a consideration of means of cooperation between the Committee and this important federal agency. It is hoped that mutual exchange of information and advice will result in real advantage to science and mathematics teachers everywhere. At least one joint meeting annually of the Cooperative Committee and the Science Committee of the Office of Education is planned for the purpose of evaluating the activities of both groups, discussion of problems of mutual interest and the cooperative planning of future projects.

A subcommittee of the Cooperative Committee has been appointed to study the problem of the development of standards for science rooms and their furnishings in the elementary and secondary schools. Some suggestions on mathematics classrooms and joint sharing of equipment in smaller schools should also be forthcoming from this group.

The Committee encourages the setting up of summer workshops in the sciences and mathematics, and suggests that these workshops might be sponsored either by educational institutions or by industry. There is a need for a joint workshop for science and mathematics teachers. With regard to workshops mathematics has a much better record than science.

Attention has been called to the rather frequent publication in European countries of "pocket" editions of science texts. The desirability of a similar practice in this country has been pointed out and it is suggested that such publications might be encouraged or even carried out by non-profit organizations as A.A.A.S. and Science Service.

A representative of the United States Office of Education has suggested the desirability of investigation of the various tests, contests, and programs that offer scholarships, fellowships, medals, and other awards. The purpose of such an investigation would be to determine their effectiveness in finding and developing the scientific talents in the youth of our nation.

It is hoped that this paper, in the nature of a report to the members of The National Council of Teachers of Mathematics on the activities of the Cooperative Committee, will assist in the implementation of the work of the Committee by calling the attention of a larger group of teachers to the Committee's recommendations. While no attempt is made to review the various reports of the Committee, the bibliography at the end of this paper will make it possible for the interested teacher to make himself more familiar with those reports.

In this brief review of the Cooperative Committee's program it does not seem inappropriate to make brief reference to a few of the previously published recommendations of the Committee which appear to be of special interest and timeliness in 1950. A much more comprehensive report of these recommendations may be found in the Schorling report in the *American Mathematical Monthly*, or the original reports especially Vol. IV of *Science and Public Policy*. It must be recognized that in a review and limited selection of the recommendations of a Committee, especially by one who was not a member of the Committee when these points were discussed, there is some danger of improper emphasis and inaccurate interpretation.

In all of its recommendations the Cooperative Committee has emphasized that science and mathematics both in the elementary and secondary schools must serve two functions, namely, to contribute to the general education of all students and to provide a dependable foundation in mathematics for future scientists and other specialists. The Committee lends its

endorsement to the list of 28 essentials for functional competence, prepared by the Commission on Post-War Plans, and recognizes the resulting improvement in mathematics for general education of the past few years. That we have no adequate means for early identification of the talented has been pointed out as a challenge to schools everywhere. While the Committee can point to progress in the use of the laboratory method in mathematics classes at all levels and in mee ing the challenge of E. H. Moore of a half-century ago, it does recommend that a scientific study be undertaken of the needs of students of scientific and mathematical promise so that a new course may be designed if such seems desirable. The desirability of relating much more closely mathematics instruction to all science taught in the high schools has been frequently emphasized by the Committee.

The Committee recognizes that many of the present short-comings in teaching science and mathematics stem from lack of adequate financial support for public education. This is reflected in inadequate pay for teachers and lack of equipment for proper laboratory teaching of science and mathematics. Furthermore an inservice program for teachers requires funds which are often not available. It is recognized that a good job of course revision cannot be done at the end of a school day by a group of tired teachers and it is suggested that teachers be employed during the summer for this purpose. While these are not new ideas by any means, the fact that attention is forcefully called to them from a new quarter and that the call will reach new listeners is encouraging and of significance. The Committee also has given complete endorsement to federal subsistence scholarships for collegiate training for the gifted.

The Committee has attributed part of the faults in mathematics teaching to our failure to take the pupil where he is and to provide for normal growth and development, again not a new idea, but one which has not always been accepted by those most apt to listen to the Cooperative Committee, and also one that can be scarcely overemphasized. Schools have been called upon to study their over-all programs with a view to better bridging the gaps in the total mathematics sequence. This need is most marked in many schools in the transition from the sixth to the seventh grade, but in some it is apt to occur between any two grades, if there is no carefully prepared course of study and inadequate supervision. It might also be noted that it may, and in fact frequently does, occur between algebra and plane geometry where these courses are organized in terms of sharp divisions of subject matter.

The Committee has observed with some concern that too often in courses both for general education and for foundation for later work in science and mathematics a wrong emphasis is placed. Too frequently the emphasis is on fact and formula and not on the scientific approach to problems and real understanding in mathematics.

At present there is no agency in science or mathematics that is systematically collecting information about present practices and present achievement so that recommendations for sensible solutions may evolve. While it is wholly to be desired, as the Cooperative Committee recommends, that some national agency be established with funds sufficient to do this work, it is necessary in the meantime that professional groups in the science and mathematics area do start some systematic collection of information upon which scientific thinking on our common teaching problems may be based. This offers a real challenge to a growing National Council of Teachers of Mathematics.

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# Miscellanea—Mathematical, Historical, Pedagogical

*By* PHILLIP S. JONES
*University of Michigan, Ann Arbor, Michigan*

### TOPOLOGY IN THE LIMELIGHT

THE ARTICLE "Topology" by Albert W. Tucker and Herbert S. Bailey, Jr. in the January 1950 issue (pp. 18–24) of *Scientific American* is the most recent of many attempts to popularize that fascinating subject. More technical but still readable is "The Theory of Braids" by Emil Artin in the Winter (1950) issue of the *American Scientist* (vol. 38, pp. 112–119).

At least one fiction writer, William Hazlitt Upson, has made money out of one topological curiosity, the Möbius strip. Mr. Upson published "A. Botts and the Möbius Strip" in the *Saturday Evening Post* for December 1945 (vol. 218, p. 22ff) and now Frank Hawthorne of Hofstra College calls our attention to "Paul Bunyan vs. the Conveyor Belt," a second article by this author using for its subject the same mathematical oddity. This latter appeared in the *Ford Times* for July 1949 which is available, free, from the Ford Motor Company, 3000 Schaefer Road, Dearborn, Michigan.

Perhaps you readers know of other places in fiction in which mathematical concepts play a key role. If so, why not send them in and share such interesting references with other teachers?

### LITERARY ALLUSIONS DEPARTMENT

Willis L. Pickard of State Teachers College, New Paltz, New York, sends us the following quotation from Aristotle, *On Man in the Universe* [*Metaphysics*], Book II:

> "The study of truth requires a study of causes. The causes through which everything comes to pass are four in number, not an infinite series."

In this connection we might note the last and posthumously published work of the famous English historian of Greek mathematics Sir T. L. Heath. This is *Mathematics in Aristotle* (Oxford University Press), which appeared in 1949. The entire 291 page book is filled with quotations and brief commentaries. For example on page 201 is to be found the following also quoted from Aristotle's *Metaphysics:* "For the chiefest forms of the beautiful are orderly arrangement, symmetry, and definiteness, and the mathematical sciences have these characters in the highest degree." It is also of interest to note (p. 246ff.) that the paradox in which circles of different radii are made to appear to have the same circumference had its origin with Aristotle although Heron, Cardan, and Galilei are among those who discussed it later. In modern terms this paradox reasons that if an automobile tire rolls in a gutter in such a fashion that the edge of the rim on which it is mounted rolls on the top of the curb then the outer circumferences of the tire and of the rim must be the same. This appears to follow because in one revolution of the wheel the circumference of the tire would be represented by the distance it passed over on the road and the circumference of the rim would be the corresponding distance passed over by it on the curb. These two distances would be parallel and equal.

It is only natural that the literary works of Christopher Morley, the son of a well known American mathematician, should be generously sprinkled with mathematical allusions. For instance in *The Trojan Horse* we find a description of the activities in a road house frequented by both Greeks and Trojans between battles as follows:

"... the accepted etiquette is that both Greeks and Trojans, even in the turn or tangency of the dance, disregard each other and make no attempt to mix. ... In the jumbled hazards of rotation

the weaving pairs approach and reverse, gracefully veer and pass, asymptote and recede. At successive cusps of an irregular epicycloid swings agile odd Pandarus, . . . "(1).

### More Pi

The note about the computation of *pi* to 808 decimal places which appeared in this department in the March Mathematics Teacher had scarcely been written when the *Scientific American* announced that Eniac, electronic computer at the Army's Ballistic Research Laboratories at Aberdeen, Maryland, had computed *pi* to 2040 places in less than 24 hours (2). A letter from D. J. X. Montgomery later explained that "Eniac computers undertook their task on their own time, with a view toward obtaining a statistical measure of the randomness of distribution of the digits of *pi* (and *e* as well) following a suggestion of Professor J. von Neumann." Actually the computation required 70 hours of machine running time and extended the value of *pi* to 2035 certain figures and 2037 probable (3).

### Pedagogical Parallel

Norman Anning of the University of Michigan gives us the above subhead and the following note:

In order to reduce 4 rods, 3 yards, 2 feet, 1 inch to inches one can reach the answer in two ways:

$$4(198)+3(36)+2(12)+1=925.$$

$$12[3\{(5\tfrac{1}{2})4+3\}+2]+1=925.$$

Probably both of these methods were taught to our grandfathers. Certainly the method of successive multiplications and additions was taught to them. We can use these to illustrate the poorer and better ways of evaluating $4x^3+3x^2+2x+1$ when $x=6$. Thus, the poorer, $4(216)+3(36)+2(6)+1=985$. and the better,

$$6[6\{6(4)+3\}+2]+1$$

$$=6[6\{27\}+2]+1=6[164]+1=985.$$

This second method is akin to turning rods into yards, then yards into feet and so on. That it is nothing more than the use of the Remainder Theorem becomes obvious when we write it

| | | | | |
|---|---|---|---|---|
| 6 | 4 | 3 | 2 | 1 |
| | | 24 | 162 | 984 |
| | 4 | 27 | 164 | 985 |

The rather tenuous moral of this story is probably due to Poor Richard: One at a time is good fishing.

### Mathematics in Scouting

Donovan Johnson of the University of Minnesota calls our attention to a one page article in pictures titled "Let's Measure It" by Glenn A. Wagner which appeared on page 24 of *Boys Life* for October, 1949. Four easy laboratory projects are suggested involving measuring a height, a bearing, a distance, and an angle of elevation. The latter is done by means of a hypsometer ingeniously made from a tobacco can. This article in a magazine published by the Boy Scouts of America suggests that perhaps not all mathematics teachers have capitalized on the connections which can be established with Scouting. Perhaps someone familiar with Scouting will send us an analysis of such connections. For example, the *Merit Badge Pamphlet Series* includes booklets on *Astronomy*, *Mechanical Drawing*, and *Weather* which contain requirements and projects involving measurement, scale drawing, interpretation of charts, graphs, and maps. These pamphlets might well be included in the classroom library.

### Internationalism and Mathematics

The first International Congress of Mathematicians since 1936 is to be held at Harvard University from August 30 to September 6, 1950. This is significant in many ways. The first International Congress was held in Chicago in 1893 in connection with the World's Columbian Exposition. This Congress at which Felix

Klein represented Germany did much to stimulate mathematical development in this country. The 1950 Congress will be the first return of the Congress to the United States. Congresses were held each four years from 1924 through 1936. The 1940 congress would have been held at Harvard had not the war intervened.

Further, this is to be a truly international congress with representatives from all countries invited and welcome. This is in contrast to the first congress held after World War I. This latter congress excluded representatives from the central European powers. It was largely planned by the International Mathematical Union which was organized in Brussels in 1919. The Union also excluded these nations. It was largely the opposition of Americans and American mathematicians to this policy of exclusion which caused the 1924 Congress to be held in Toronto rather than in this country as originally planned, and which led to the eventual death of the Union.

The American Policy Committee for Mathematics is now cooperating with committees in other countries planning the formation of a new International Mathematical Union.

One of the seven sections of the Harvard Congress will be devoted to the History and Pedagogy of Mathematics with an invited lecturer in each of these fields and with a number of shorter reports or papers. The final program of the section had not been completed at the time this was written (February 12, 1950).

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(1) Christopher Morley, *The Trojan Horse*. (New York: J. B. Lippincott Company, 1937) pp. 87, 88.
(2) *Scientific American*, vol. 181 (Dec., 1949) p. 30.
(3) *Scientific American*, vol. 182 (Feb. 1950) p. 2.

For a complete discussion see George W Reitwiesner, "An ENIAC Determination of $\pi$ and $e$ to more than 2000 Decimal Places." *Mathematical Tables and Other Aids to Computation*, vol. IV (Jan. 1950), pp. 11–15.

## A Note on $\pi$

An article in the March, 1950 Mathematics Teacher (page 120) in commenting on "What's new about $\pi$" calls attention to the recent publication of the value of $\pi$ to 808 decimal places. It may be of minor interest to note that in the January 1950 issue of *Mathematical Tables and Other Aids to Computation* (page 14) the value of $\pi$ is given rounded off to 2035 decimals. In the same place is found the value of $e$ to 2010 decimal places.

The article includes a description of the methods used and confirms the 808 decimal place value of $\pi$ computed a few years ago.

Cecil B. Read, University of Wichita, Wichita, Kansas

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THE enthusiastic comments which were made by those who attended our first Institute held at Wellesley College in 1949 have encouraged the Association to sponsor a similar gathering in 1950. The Institute is designed for teachers of mathematics in elementary schools, secondary schools and colleges. Its purpose is to keep teachers up-to-date on the mathematics they teach, to increase their appreciation of the subject, to give them greater knowledge of its application through first-hand contact with top-notch men (and women!) in industry and other fields, to make it possible for them to share their ideas concerning methods used in teaching, to acquaint them with mathematical activities near Boston, and to enable them to relax among congenial and friendly surroundings.

Aids to the teaching of Mathematics will be on display, including latest texts and professional books, mathematical puzzles and recreations, mathematical instruments, calculating devices, models and devices made by teachers and pupils.

The mathematics laboratory will provide opportunity for teachers to construct mathematical devices and teaching aids for use in their classrooms. There will be a small fee to cover the cost.

## PROGRAM

*Tuesday, August 22*

1:00–5:00 P.M.—Registration.

8:00–9:30 P.M.—"Exhibits in Action." Miriam Loring, Mathematics Department, Senior High School, Belmont, Massachusetts.
"The Whoarue Problem" Florence Magner, Mathematics Department, High School of Practical Arts, Boston, Massachusetts.

*Wednesday, August 23*

9:00–10:00 A.M.—"Non-Euclidean Geometry." Garrett Birkhoff, Professor of Mathematics, Harvard University, Cambridge, Massachusetts.

10:30–11:45 A.M., 1:30–4:45 P.M.—Laboratory and Discussion Groups.

8:00–9:30 P.M.—"The Use of Instructional Materials in an Arithmetic Program." Foster E. Grossnickle, State Teachers College, Jersey City, New Jersey.
"Building Number Concepts." Prof. F. Lynwood Wren, Peabody Teachers College, Nashville, Tennessee.

*Thursday, August 24*

9:00–10:00 A.M.—"Nuclear Physics—A Study in Extremes." Dr. Jerold R. Zacharias, Director of Laboratory for Nuclear Science and Engineering, Massachusetts Institute of Technology, Massachusetts.

10:30–11:45 A.M., 1:30–4:45 P.M.—Laboratory and Discussion Groups.

5:00–9:00 P.M.—Picnic.

*Friday, August 25*

9:00 A.M.–12:00 M.—Visit to place of mathematical interest.

1:30–4:45 P.M.—Laboratory and Discussion Groups.

8:00–9:00 P.M.—"Taking the 'Numb' out of Number." Harold B. Gores, Superintendent of Schools, Newton, Massachusetts.

*Saturday, August 26*

9:00–10 A.M.

10:00–11:45 A.M., 1:30–4:45 P.M.—Laboratory and Discussion Groups.

8:00–9:00 P.M.—"Logarithms and Ivory Towers—Thoughts on Art, Mathematics and Life." Helen C. Miles, Art Supervisor, Newton, Massachusetts.

*Sunday, August 27*

A definite program for your enjoyment is planned for afternoon and evening.

*Monday, August 28*

9:00–10:00 A.M.—"Substitution Ciphers—A Method in Cryptanalysis." Dr. Ruth G. Simond, Professor of Mathematics, University of Vermont, Burlington, Vermont.
10:30–11:45 A.M.—Laboratory and Discussion Groups.
1:15–5:30 P.M.—Visit to place of interest for mathematicians.
8:00–9:00 P.M.—"Determinants and Grassman Algebra." Prof. Saunders MacLane, University of Chicago, Chicago, Illinois.

*Tuesday, August 29*

9:00–10:00 A.M.—"A Mathematician Looks at Language, or Is English Really Necessary?" Bruce Billings, Director of Research, Baird Associates, Inc., Cambridge, Massachusetts.
10:30–11:45 A.M., 1:30–4:45 P.M.—Laboratory and Discussion Groups.
6:30–9:30 P.M.—Banquet.

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Discussion Groups will include:

1. "The Use of Applications from Science in the Teaching of Algebra and Geometry."
   Leader: Henry W. Syer, Assistant Professor of Education, Boston University, Boston, Massachusetts.
2. "Non-Academic Mathematics in High School."
   Leader: Howard F. Fehr, Professor of Mathematics, Teachers College, Columbia University, New York.
3. "Statistics in the Twelfth Grade and in the Freshman Year of College."
   Leader: Elmer B. Mode, Professor of Mathematics, Boston University, Boston, Massachusetts.
4. "Aids in the Study of Algebra."
   Leader: Frances Johnson, Mathematics Department, High School, Oneonta, New York.
5. "Basic Meanings in Arithmetic."
   Leader: Professor F. Lynwood Wren, Peabody Teachers College, Nashville, Tennessee.
6. "Originals in Algebra and Geometry."
   Leader: Charles H. Mergendahl, Head of Mathematics Department, High School, Newton, Massachusetts.
7. "Beginner's Algebra with Fewer Rules and Better Reasons."
   Leader: William R. Ransom, Professor of Mathematics, Tufts College, Medford, Massachusetts.
8. Laboratory—
   Robert C. Yates, Lieutenant Colonel, U. S. A., Mathematics Department, U. S. Military Academy, West Point, New York.
   George Kayes, State Teachers College, Montclair, New Jersey.
   Christina S. Little, Acting Head of Mathematics Department, Girls' High School, Boston, Massachusetts.
   Elwood M. Stoddard, Mathematics Department, Hingham, Massachusetts

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The only fee will be $10.00 for registration and the small laboratory charge for materials if you join that group. Room and board on the Tufts campus will be available at reasonable rates, but reservations should be made before June 1 if you wish to be sure of accommodations.

For any additional information, write to

Janet S. Height
41 Richardson Avenue
Wakefield, Massachusetts

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With profound regret we announce the death on April 22 of Professor Raleigh Schorling, of the University of Michigan. Long a leader in the Council and in mathematical education, he will be missed by many friends. We extend heartfelt sympathy to his family.

# Group Affiliation with the National Council

*By* H. W. Charlesworth

*Chairman of Affiliated Groups, East High School, Denver, Colorado*

The school year is about over. We have had another very successful year for the National Council. Much progress has been made. We have been blessed with capable and untiring leadership. Conventions have been better than ever, more numerous and more scattered. Our memberships have been steadily increasing. New activities have been started, among these being the Delegate Assembly which had its initial meeting in Chicago in April. Looking back over these recent accomplishments gives us encouragement to go on and makes us anxious to be a part of future developments.

Looking ahead, we see the possibilities of even greater strides being made in the work of group affiliation. We see the possibility of having a state council of mathematics teachers in ten or twelve more states within another year. Last fall twelve states were organizing or planning to organize a state group. Eight of these are now affiliated with the National Council. By the end of another year, it is probable that we shall have at least thirty-six well organized and functioning state councils of mathematics teachers.

We are looking toward the time when we shall have a state council in virtually every state. A few larger states may have more than one such group in the state, smaller or less populated states may combine into a bi-state, tri-state, or regional organization. Smaller groups within the state, district groups, county groups, city groups, will associate themselves with the state group. The state group in turn will affiliate with the National Council. Under such a plan, state groups will be the only ones directly affiliated with the National Council. The state groups should provide means whereby county, district, and city groups might associate themselves with the state group. The state council should make this provision clear to smaller groups in the state and should endeavor to get them to associate themselves with the state council. Officers of state councils would do well to make this a part of their program of activities for this next year. We want to reach all the mathematics teachers, even in remote areas. Eventually all groups that desire it will be affiliated with the National Council either directly or indirectly. Under such a plan, the National Council can be of the greatest service to mathematics teachers all over the country, reaching out through its affiliated groups and these, in turn, through their associated groups to the individual teacher wherever he may be.

What can affiliated groups be doing during the next three or four months? Although theoretically a vacation period leaders of groups can get a great deal done during these next few months. The executive committee of a group can be busy planning for the next year, not only planning a good program for the fall meeting, but organizing committee work for the coming year. After all, the real work of a group must be done by competent committees. If the executive committee plans well, arranging the time and place for meetings of committees, setting due dates for reports, etc., much more can be accomplished by the group during the year. Do some long-range planning for your group. There is another thing that might be done this summer, or later. Representatives from several state groups within a certain area might get together, such as is being done by five or six state groups of the region during the summer convention in Madison, Wisconsin, next August. Officers and others of these five or six groups will meet to exchange ideas and to lay plans for coordinating their efforts on common endeavors. This kind of cooperation should be helpful to all.

What are the further possibilities of the Delegate Assembly? At their first meeting in Chicago in April, the time was consumed with matters of affiliation. Subsequent meetings should be concerned with other matters as well. It is hoped that this body will meet annually and become, to some extent, a policy forming body for the National Council. The Delegate Assembly could be of great help to the Board of Directors, in suggesting changes in old policies and in recommending new policies. Certainly, there should be some means by which the members of the National Council could initiate thought on matters of policy, perhaps the Delegate Assembly is the means.

My work over the past year or so as chairman of affiliated groups has been very interesting and very encouraging. It has taken a great deal of time, but the results have been gratifying. I have had the help and cooperation of many people. To these I wish to express my sincere thanks. The state representatives have been a great help. I have called upon them for help several times during the past months and have had excellent response. Their help means much to the National Council.

We can look ahead, then, to still more improvements in affiliation matters next year. States which have not yet taken the initial steps toward the organizing of a state council of mathematics teachers are advised to do so soon. It is assured that group affiliation with the National Council will not be a temporary thing. This work will go on and all possible effort will be put forth to make continuous improvements. If I can be of help to you in your efforts to organize a state council, please write to me. I shall do everything I can to help.


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# Have You Heard?

*By* EDITH WOOLSEY
*Sanford Junior High School, Minneapolis, Minn.*

THIS is the last issue of THE MATHEMATICS TEACHER for this school year. Suppose we look ahead and see what the Council has in store for us. On July 3 there will be a meeting in St. Louis in cooperation with the N.E.A. Prof. C. H. Butler, of Western Michigan College of Education in Kalamazoo, is preparing the program. Miss Ella Marth, of Harris Teachers College in St. Louis, is the local chairman. With these people in charge, the program will surely be excellent. If you attend it, you will also have the opportunity of going to the N.E.A. meetings and of seeing the city itself. St. Louis is full of places and things of historic interest. Have you seen the Eads bridge? That made bridge history as well as United States history. It was completed about 1870 and was the first railroad bridge across the Mississippi river that far north. It made it possible for the railroad to cross the river and go on out west, which resulted in the eventual disappearance of the covered wagon. The Lindbergh museum is there, too, and Shaw's gardens, and many other places worth seeing.

In August the Council is having a convention in Madison at the University of Wisconsin. Dr. John Mayor is in charge of the program and is planning an excellent one. The meetings will be devoted to the problems of mathematics teachers in various fields: elementary arithmetic, junior high, senior high, junior college, teacher training, visual aids, use of community resources, business applications, evaluation, guidance,—all this and more, too.

In addition to the fine program of meetings, there is a well planned program of social events. The dates of this convention are August 21, 22, 23 and 24. On Monday evening, August 21, there will be a boat trip to Picnic Point where a delicious picnic supper will be served. On Tuesday there will be a discussion dinner on the terrace, just outside the Union, overlooking beautiful Lake Mendota. On Thursday, after the meetings are over, a trip to points of interest in Wisconsin will start at 9:30 A.M. If you have driven through the state you know how beautiful it is. If you have not had this experience, then take advantage of this opportunity.

Perhaps you would like to know more about the city of Madison. It is distinguished for having both the State University and the State Capital. It has a population of about 100,000. The city is located 139 miles northwest of Chicago, 275 miles southeast of St. Paul. It is built upon a hilly isthmus between two lakes. The University has a 1000 acre campus on the shore of Lake Mendota. Madison was named after President James Madison. It became the capital of Wisconsin Territory in 1836 and the state capital in 1848. It is reached by three railroads: Chicago, Milwaukee, St. Paul and Pacific, Chicago and Northwestern, and the Illinois Central. Many highways go through Madison: U. S. highways numbers 12, 14, 18, 51, and 151, and state highways 13, 19, 30, and 113. Madison can boast about its $10,000,000 airport of 875 acres with regular service to Chicago—Seattle—New York. Can you think of a more delightful or more profitable way to spend four days of your vacation than to attend this convention in Madison?

The winter meeting of the Council is to be at the University of Florida in Gainsville, Florida, on December 27, 28, and 29, in conjunction with sessions of the Mathematical Association of America and the American Mathematical Society. The program chairman is Prof. James H. Zant of Oklahoma A. and M. College, Stillwater, Oklahoma. The program promises to be outstanding.

# AIDS TO TEACHING

*By*

HENRY W. SYER
*School of Education*
*Boston University*
*Boston, Massachusetts*

DONOVAN A. JOHNSON
*College of Education*
*University of Minnesota*
*Minneapolis, Minnesota*

## BOOKLETS

*B.32—Time Telling*

Educational Service Department, Hamilton Watch Company, Lancaster, Pennsylvania.

Unit; Teachers Manual, 16 pages; pupil lesson sheet, 4 pages; chart; free.

*Description*: This unit was developed by Herbert S. Zim, science consultant, and Dr. Morris Meister, a high school principal. It describes in a direct and interesting manner the development of time measurement and the problems in watch manufacturing. It explains the astronomical basis for time measurement with interesting sidelights on the checking of time measurement by observatories. The place of time measurement in our scientific world as well as the contribution of science to watch making is concisely discussed. It explains how a watch works as well as the use of scientific principles involved in the pendulum, expansion due to heat, friction, and energy. In addition to the measurement of time, the use of gear ratios and accuracy of manufactured parts are more mathematical topics. The unit is completed by listing suggested activities for pupils, books on time to read, and how to use the student lesson sheets. A large chart to be used in teaching the unit is included. The lesson sheets include a multiple choice test.

*Appraisal*: This unit is well written by educators who understand the needs of classroom teachers. It has a great number of facts scientific, sociological, and mathematical packed into its few pages and yet the story has continuity and interest. The Hamilton Watch Company is to be commended for the small amount of advertising which is entirely confined to pages exclusive of the unit story. The unit is most suitable for a science class since there are so many scientific concepts involved. However, the mathematics teacher will find the material on the development of time, the emphasis on accuracy of measurement, and the gear ratios as being excellent supplementary material for teachers.

## CHARTS

*C. 14—Aiken Algebra Charts*

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24 charts, each 49″×38″; complete set with tripod stand, $43.60.

*Description*: The twenty-four charts are printed on a good quality dull-finish paper, reinforced on the back around the edges with cloth tape one-half inch wide. They are mounted on a sturdy tripod that may be folded for storing, and permits charts to be raised or lowered in classwork.

The charts are printed in black and red ink carefully planned to attract the eye. In twelve of the charts a blackboard is pictured, and questions, answers, and problems are printed in script writing in white on black. These are especially sharp.

Charts 1–5 deal with customary introductory material of first-year algebra: using letters to represent numbers, order of operations, vocabulary and evaluation.

Charts 6–9 present signed numbers and the laws governing their use. Positive numbers are represented by black lines and diagrams, negative by red.

Charts 10–17, or one-third of the whole, deal with simple equations and their solution. Charts 12 and 13 use horizontal addition and subtraction, whereas chart 16 uses vertical. Chart 16 is devoted to the solution of two-step problems, and Chart 17 to the solution of fractional equations. Chart 24 introduces the quadratic formula and its use.

Chart 18, dealing with the removal of parentheses, is very good. It is followed by Chart 19, the only chart on factoring, which is a good lesson in process of finding common monomial, but makes only hurried reference to binomial factors.

Charts 20–23 introduce the miscellaneous topics of exponents, the coordinate system, the Pythagorean theorem, and roots and radicals. Chart 22 on the Pythagorean theorem with little boys in mantles and sandals teaching the lesson, and the rule printed on an unrolled scroll, is one of the best.

*Appraisal:* The charts are excellent for introduction of new material, and are equally good for review. The presentation is conventional enough to fit most textbooks. The charts are not interdependent, so the teacher can make full use of them no matter what order she used in her own class. The language is well-chosen and appropriate.

One reviewer feels that the material on most of the charts is crowded with important material printed too small to be read by children sitting in back of the room. She says this is especially true of exponents in Charts 2 and 20, and of statements in balloons throughout all charts. The other reviewer disagrees with this opinion.

The first twenty charts (or twenty-one, depending upon time teacher introduces graphing) deal with foundation work laid in Grade 8 and first semester of Grade 9. The series would be more representative of first-year algebra if there were charts presenting work in verbal problems, simultaneous equations, ratio and proportion, and trigonometric functions.

The following comments came from two classes of students in which the charts were tried: "The whole chart simplifies algebra," "It's babyish," "The modern way of illustration makes the chart attractive," "The figures are very realistic," "The attitude of the animals and their effort to do work gives impetus to us," "Each page arouses our curiosity as to what is coming," "The cartoons are stupid and silly," "We want human figures—not animals," "The charts make the room look like a primary grade," "Use the little men all the way through—they're easier to get used to than monkeys," "The fundamental processes are clearly and simply demonstrated," "It shows only the obvious easy things," "Each section is thoroughly explained," "There's not enough factoring—it's all on one sheet, and not explained," "The colors are vivid in contrast," "We don't like the red and black, it all blurs as you sit looking at it."

Generally speaking, the better students did not seem to care for the set and considered it an insult to their mentality. Some said if it were on the style of the science charts—purely technical and unadorned, it would be more acceptable to them. The more ordinary pupils took to it, and appeared to find assistance in it.

The chart on similar terms, their assorting and collecting is excellent. The one on signed numbers is a fine piece of coordination with the History and General Science work of the first year, bringing in the airplane invention of 1903, the atom bomb 1945, the thermometer, jet plane and parachute. Showing a graph on an early page is good. The idea of its usefulness can be more easily instilled.

The chart on subtraction of signed numbers which endeavors to show change of direction by means of a cat and dog being pulled apart seems very far-fetched and makes little sense to the youngsters. The laws of multiplication are pictured by a

series of levers. The sign in the answer in each law is explained as the result of a rotational effect. One reviewer thought this was understandable and effective; the other said that since it was based on positive and negative rotation, it would mean nothing to freshmen who had never had physics.

On the chart "Roots of Equations," there is a chance for confusion, since 2 is tried as a root but proves not to be. The incorrectness is not made emphatic enough to prevent confusion.

In the chart on solving fractional equations, step one is bracketing of both sides; step two, multiplication by the L.C.D. and step three the removal of the brackets. Why introduce two unnecessary and difficult steps?

The exponents are well handled, but no mention is made of fractional exponents. Couldn't these be included on the Root chart, at least?

There is far too little factoring. The axioms are done in detail, one page to each. But factoring is hardly mentioned. One reviewer feels it is a deplorable omission since factoring is the main theme in a first-year algebra course.

An eight-page booklet entitled "Aiken Algebra Charts with Cartoons" may be obtained from A. J. Nystrom and Company. It contains a miniature of each of the 24 charts and will give the prospective buyer another way of judging their value. (Reviewed by M. Isabella Savides, Warren Junior High School, Newton, Massachusetts; and Sister Ernestine Marie, Ryan Memorial High School, Dorchester, Massachusetts).

## EQUIPMENT

*E. 31—Addit; E. 32—Tallyit*

The Maxim Games Company, 514 West 113th Street, New York, N. Y.

Games; 55 cards and 55 discs; each $2.00.

*Description of E 31–32:* Each card of the game Addit contains two one-digit numbers to be added while the discs contain the answer to these sums. Addit may be played by two or more players. The object of the game is to win as many cards as possible. A card is won when the sum on a card face up in front of a player is correctly matched with a disc. The cards and discs are turned up on a certain sequence by the players to add interest to the game.

Each card of the game Tallyit contains two one-digit numbers to be multiplied while the discs contain the products of their multiplications. Otherwise, Tallyit may be played in the same manner as Addit.

*Appraisal of E.31–32:* These educational games are attractively printed on fairly durable material. Each deck is printed in two colors, so that by using the cards of a certain color alone an easier game may be played than when using the entire deck. There is no reason why cards cannot be selected according to the pupils needs and interests of the color of the cards. Although the cards are advertised as suitable for age 6 to 60 it is doubtful that pupils beyond sixth grade will be interested. These games will provide the teacher with material to make drill on addition and multiplication more interesting. The games may be played without the direct participation of the teacher. They could be particularly suitable for parents to use to participate in their child's learning. However, the cost of the game is excessive, but there is no reason why the resourceful teacher cannot have her pupils make cards and discs for a similar game. The same idea can also be applied to practice on subtraction and division.

## FILMS

*F. 51—Caught Mapping*

General Motors Corporation; Dept. of Public Relations, 1775 Broadway, New York, N. Y.

16 mm. sound film; black and white; 400 feet; 10 min; free (for payment of postage).

*Description:* This film begins by show-

ing us handmade maps and then a modern printing plant making road maps. After convincing us that cars use such maps and find them essential in planning and taking trips we are given an insight into the way such maps are kept up-to-date. Road scouts keep traveling constantly to find the changes that need to be made, checking altitudes and the grade of the road. This latter measurement uses an instrument called the gradometer. Finally the overlay method of revising maps which saves the labor of redrawing the whole map is shown.

*Appraisal:* Here is a specialized branch of surveying which is based very directly upon mathematics. The film was not produced as a teaching film, but as an advertising film for General Motors, and therefore shows G. M. cars at every opportunity. They are not obstrusively or objectionally predominant, and do not detract from the use of the film at all.

The mathematical ideas which should develop in discussion after seeing this film are essentially very simple and fundamental: distance, direction, scale drawing, altitude, grade (or slope) of a hill, units of measurement, degree of accuracy, and dependence of pleasure driving and commerce upon the mathematical accuracy of the maps. These make the film appropriate for junior high school level classes, but would be of interest throughout the high school as well. The reality of the scenes and the automatic assumption by the surveyors of the usefulness of mathematics are the most important features of this film.

## FILMSTRIPS

*FS. 69—Arithmetic of Algebra*

The Jam Handy Organization, 2900 East Grand Blvd., Detroit, Michigan.

1943, 35 mm. filmstrip; black and white; silent; 45 frames; $4.00

*Description:* This filmstrip begins by illustrating that in arithmetic we add, subtract, multiply, and divide numbers representing many things such as apples, men, pounds, airplanes, etc. A picture of an apple showing its various parts emphasizes that the arithmetic in the solution of a basic problem is the same whether we are interested in the parts of an apple or in the *whole* apple. The variable quantity used in arithmetic problems when written in letters becomes an *algebraic* expression. Examples are given to show the following: (1) the rules for addition, subtraction, multiplication and division, and for the use of exponents apply to variables as well as numbers; (2) unlike quantities cannot be added by adding their coefficients, (3) multiplication of combinations of quantities; (4) division of quantities; (5) cancellation; and (6) finding the factor of a quantity. The filmstrip ends by summarizing the principles learned in arithmetic which will apply in the same way to quantities and variables.

*Appraisal:* This filmstrip is not meaningful to beginning classes in algebra until they are familiar with the various concepts which are presented. Many teachers will object to frames 32–34 dealing with cancellation. The most effective part is the emphasis placed on the fact that the basic principles learned in arithmetic can be used in like manner to quantities and variables in algebra. Most of the material presented might be illustrated just as well on the blackboard. (Reviewed by Ida May Bernhard, San Marcos Public Schools, San Marcos, Texas.)

*FS. 70—Equations and Formulas*

The Jam Handy Organization, 2900 East Grand Blvd., Detroit, Michigan.

1943, 35 mm. filmstrip; black and white; silent; 63 frames; $4.00

*Description:* This filmstrip considers the following four topics: "What Are Fquations?" "Manipulation of Equations," "Types of Equations," and "Formulas." Frames 1–9 give illustrations of statements of fact heard in everyday conversa-

tion. They show these statements can be written in a kind of shorthand to form a simple equation. Frames 10–29 emphasize the importance of maintaining the two sides of an equation in balance. Pictures of a set of scales balanced with weights show how the same amount can be added to each side or subtracted from each side without disturbing the balance. Also, the scales keep their balance when each side is multiplied or divided by the same number. The problem $X+4+10=20$ is pictured on the scales and then solved. Frames 30–41 show a simple equation of three unknowns to be solved simultaneously; quadratic equations, and quadratic with two unknowns. Using two examples, frames 42–61 stress the value of the formula. First, the problem of how much gasoline is needed for a three-quarter hour flight is worked out by substitution in a formula. Next the formula for finding the total lift of an airplane is solved in terms of the wing area. The last two frames summarize the points to remember in the filmstrip.

*Appraisal:* The material presented covers too great a range for purposeful teaching. Frames 3–9 and 14–29 may be used in a first year algebra class for developing a better understanding of concepts and for review purposes. The remainder of the filmstrip is confusing to the class. In the opinion of the reviewer certain terms and expressions used are not conducive to good teaching. Some of these are: "juggling the equations" (frame 29); "cross-multiplication" (frame 32); "we can flip this formula around to suit" (frame 58); and, "any formula can be changed around to give any one of the letters values" (frame 61). The solution of the formula in frames 57–61 is confusing. This filmstrip failed to arouse interest in either first or second year algebra classes. (Reviewed by Ida May Bernhard, San Marcos Public Schools, San Marcos, Texas.)

*FS. 71—Areas by Integration Using Polar Coordinates*

Society for Visual Education, 100 East Ohio Street, Chicago, Illinois.

35 mm. filmstrip; black and white; silent; 34 frames; collaborator: Edwin A. Whitman; Carnegie Institute of Technology, Pittsburgh, Pennsylvania; $4.00.

*Description:* There are four frames of general discussion and titles, seven on the theory of the integrals, thirteen on single integrals, and ten on double integrals using polar coordinates. Each series builds the necessary diagram in a sequence of steps which shows the elements which make up the sum whose limit is the integral.

*Appraisal:* Many of the diagrams of higher mathematics are difficult to draw accurately while describing and explaining the process. Although poorly sketched ones, adapted by the instructor to his own particular method of development, and to the classes own peculiar brand of misunderstandings and questions will always have a use in teaching calculus, there is a great need for accurate drawings which progress from the simple beginning to the complete drawing with some amount of speed. This filmstrip is especially useful for review of the method used to set up single or double integrals with polar coordinates.

## INSTRUMENTS

*I. 20—Yoder-Lafayette Bubble Attachment* (No. 11 and 12)

Yoder Instrument Company, East Palestine, Ohio.

No. 11—Yoder-Lafayette Bubble Attachment; for day light use only; screw driver; in cardboard box; each $12.00.

No. 12—Yoder-Lafayette Bubble Attachment with Radium illuminated bubble; for day or night use; screw driver; in cardboard box; each $15.00.

*Description:* The Yoder-Lafayette Attachment is made of the highest quality level and angle reflector accurately mounted in a brass tube so the observer sees the

bubble (leveled) opposite the object of comparison and in the same "field of vision." The bubble has an extension magnifier which incidentally adds to the accuracy by lengthening the sight-tube. The level sight-tube is carefully mounted in a special holder. This complete attachment is quickly and easily attached to the frame of the Yoder-Lafayette Sextant No. 10 in place of the plain tube with three brass screws. This No. 11 model is useful only in the day time.

No. 12 model is suitable for the observation of stars at night; three of the highest intensity (30 ml.) radium tubes are securely mounted over the bubble. The pure radium powder will give approximately full strength luminosity for 5 years, after which it can be replaced if found desirable.

*Appraisal:* The "Bubble Sextant" was developed primarily for aerial navigation because the "bubble" established an "artificial horizon" when the real horizon cannot be seen above the clouds. It also eliminates the necessity of making a correction for "dip" (the distance above sea level). It has not only this advantage but a Bubble Sextant may be used in place of a Plain Sextant to "shoot" (find the altitude of) the sun and stars from a ship. One may also make direct observations "inland" without establishing an artificial horizon as is necessary in using a Plain Sextant. In surveying it may be used for finding the height of objects directly where the base of the object is not on a level with the observer; to find the angles of inclination and declination, find levels and run contours.

The Bubble Attachment makes the Sextant an unusually versatile instrument for practical observations by adults, by students, or by Boy Scouts. (Reviewed by Jacqueline LeMay, Boston University, Boston, Massachusetts.)

*I. 21—Trulog Duplex Calculator*

G. Norman Albree, 33 Broad Street, Boston, Massachusetts.

Graphic Logarithm table; 6″ circle; $1.00 (ed. discount).

*Description:* This instrument is not a circular slide rule, but is a graphic table of logarithms printed in spiral form on both sides of a circular piece of cardboard. There is an arm pivoted at the center which rotates on both sides of the calculator and has a hair line for ease in reading. The A-face contains logarithms of numbers from 10,000 to 32,100 and the B-face from 31,170 to 100,000. These numbers are printed on a scale which runs above a spiral line with the corresponding logarithm scale directly below.

*Appraisal:* Although it is true that a five-place table of logarithms could not be printed on a six-inch circle, neither is this device as convenient nor as accurate as a five-place table. It is ingenious and compact, but somewhat confusing to beginners who are not used to interpolating on a logarithmic scale. The booklet which explains the uses of the calculator is written for one who knows no theory and needs therefore a naive, childish sort of explanation. The five pages of explanation give far too much space to the method of reading the scales as compared with the methods of computing with logarithms ($3\frac{1}{2}$ to $1\frac{1}{2}$).

## SOURCES FOR LABORATORY WORK

*Sl. 19—Plane Geometry Experiments*

D. Van Nostrand Company, 250 Fourth Avenue, New York 3, N. Y.

Booklet; $7\frac{5}{8}″ \times 10\frac{1}{2}″$; 106 pages; Authors: Allene Archer, Miles C. Hartley, and Veryl Schult.

*Description:* The first 78 pages of this booklet describe 60 experiments in workbook fashion with spaces to be filled in as the experiments proceed. These are permanently bound into the book, but are followed by 20 plates of geometric drawings which are perforated and can be torn out to use with the experiments as needed.

In addition, there are two heavier, cardboard sheets containing three protractors and three straightedges to be cut out and used throughout the work. The experiments cover the usual material of plane geometry: triangles, parallel lines, polygons, angles and lines connected with circles, locus, similar figures and area.

*Evaluation:* A tabulation of the materials called for to perform the experiments and the number of experiments in which they appear may give an indication of the scope and the limitation of the geometry experiments described: pencil—59, paper—34, ruler—30, straightedge—16, scissors—6, compasses—20, protractor—35, pins—29, workboard—28, colored thread—2, weight—1, card—1, and circular objects—1. This shows that the booklet gives directions for the usual kind of inductive exercise which geometry teachers have used for some time to introduce topics in their courses, but in a more systematic and uniform fashion. The choice of exercises is excellent, the directions clear, and the idea commendable. The only change suggested is that more of the experiments should have been devised and included using the material such as colored string, weights, cards and circular objects. The last named objects are used in the usual experimental determination of the value of pi. Why could not more manipulation, observation, measurement and induction be obtained from three-dimensional non-classroom objects?

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# EDITORIAL

With this issue of THE MATHEMATICS TEACHER my editorial work ceases. I want to take this opportunity to express my appreciation for the excellent support given to me for over twenty years now by members of The National Council of Teachers of Mathematics and other friends of the organization. During this time the Council has grown from a small, struggling organization to a very important force in mathematical education.

Dr. E. H. C. Hildebrandt was chosen at the recent Chicago meeting to be the new editor of the TEACHER. Mr. H. W. Charlesworth is the new president of the Council. I want to extend my congratulations to them and to the other new officers; and to all the members of the Council my best wishes for a renewed and increased success.

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Mr. H. W. Charlesworth of East High School, Denver, Colorado, will be giving an advanced course in the teaching of mathematics at the University of Colorado during the first term of the summer quarter, 1950.

# BOOK REVIEWS

*Schulreform und Mathematischer Unterricht.* By Walther Lietzmann. Quelle & Meyer, Heidelberg, 1949.

In this slender volume Professor Lietzmann, whose earlier books in the field of popularized mathematics have found many enthusiastic readers outside Germany, has published the essential content of lectures given for prospective teachers in German secondary schools at the University of Göttingen during the winter semester 1948–49. The noted Geman educator appraises the work of the mathematics teacher in secondary schools from the broad perspective of present-day educational planning and redefines the curricular position of mathematics in harmony with modern tendencies in European education. After surveying the history of educational reform since the turn of the century and taking a glance at the school systems of other countries including the U. S. A., he deals with the factors that determine effectiveness of teaching, from school administration to classroom method, from the personalities of teacher and student to matters of objectives and evaluation. The treatment is unusually succinct, sometimes to the point of irksome brevity, which was probably enforced by the economic limitations of book publishing in Germany today.

Professor Lietzmann speaks with the voice of a broad-minded conservative, who would retain what is good in innovation but avoid radical changes. In particular, there is not hint that he would favor the adoption of a one-track scheme of secondary education as advocated by certain U. S. Military Government officials in Germany. Despite all the changes brought about by two World Wars the American conception of a common high school for all pupils is making little headway in Europe.

In consequence, the mathematical fare that German secondary schools are able to set before their select clientele astounds the American teacher and may even fill him with envy. From the outline given in the appendix, one gathers that German pupils in certain secondary schools spend eight consecutive years in carefully articulated mathematics courses (with four or five class hours a week) and are carried to the heights of differential and integral calculus. One is also impressed with the soundness and thoroughness of the teaching methods which the author recommends; and last, but not least, one rejoices in the thought that German pedagogs are exposed to the views of a man who is completely free from chauvinistic bias and frequently points to the achievements of non-German schools as worthy examples. It is a bit disappointing to this reviewer that the writer nowhere touches on the place of mathematics in what we Americans call "general education"; but this lack is a predominant symptom of European education, which has made little progress in emancipating itself from premature specialization. Professor Lietzmann proposes a list of mathematical symbols for universal adoption by teachers of elementary mathematics. The list, whose appearance suggests the existence of disagreement in current German practice, differs from standard American symbolism chiefly in the decimal point, which is represented by the comma, and the division symbol, which is : rather than ÷. PAUL R. NEUREITER.

*Gudiance Handbook for Teachers.* By Frank G. Davis and Pearle S. Norris. McGraw-Hill, New York. x+344 pp.

The authors of this book believe that "guidance is and should be the function of the homeroom and classroom teacher" and that "A dozen teachers with an enthusiasm for guidance add up to more than one full-time specialist." Accordingly, this book has been written for the teacher, and seems intended to furnish him 1) a conception of the meaning of guidance as it relates to students' adjustment needs, health problems, family difficulties, learning deficiencies, and curricular and vocational decisions; 2) an understanding of what he can do, in the classroom, through extracurricular activities, by means of home visits, and in cooperation with others, both teachers and members of the community; 3) tools with which to do the job.

The last of the three above suggests the practical nature of this book. Most of it deals with specific suggestions and illustrative situations. Many useful techniques are described, such as questionnaires, autobiographies, anecdotal records, scattergrams, tests and scales, guidance clinics, counselling procedures. A large number of record forms are reproduced for teachers' use, including cumulative pupil personnel records, a capacity-achievement report, a home visit record, a pupil's plan book outline, and a health observation record. In addition, there is a bibliography of selected and annotated references at the end of each chapter for the reader who is interested in additional help.

In considering the value of this book for the mathematics teacher, several points can be made. 1) Many teachers will feel that much of the material is of use primarily to the homeroom teacher. The authors concede this to some extent, at least by implication, but maintain that far more can and should be done by the classroom teacher than is generally realized. 2) The discussions are easy to follow, the language is down-to-earth, the use of anecdotes is excellent, and most of the suggestions impress one as being both valuable and usable. 3) The book should be a real challenge to every classroom teacher who wishes to do more to help his students solve their personal problems.—CLIFFORD BEBELL.

# NEWS NOTES

## THE LOUISIANA-MISSISSIPPI BRANCH OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The twenty-seventh annual joint meeting of the Louisiana-Mississippi Section of the Mathematical Association of America, and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics was held at Centenary College in Shreveport, Louisiana, on February 24 and 25, 1950.

With Professor J. W. McClimans presiding, the following program of the Council was presented the morning of February 25.

1. Report of the Wichita meeting
   Margaret Sumrall, Sunflower Junior College
2. Broadening Horizons in Mathematics
   Margaret Rae Davis, Southeastern Louisiana College
3. An Investigation of the Diversity of School Subjects Taught by Teachers of Mathematics in Louisiana High Schools
   Mrs. Sara W. Hollingsworth, Ouachita Parish High School Monroe, Louisiana
4. Slide Rules
   Lester R. Ford, Illinois Institute of Technology
5. Simplified School Program
   M. C. Wicht, Louisiana State University
6. College Entrance Requirements in Mathematics
   L. M. Garrison, Louisiana Polytechnic Institute
7. How Mathematics Teaching May Be Improved in the Secondary School.
   W. Donald Turner, Winnfield High School

Friday afternoon Centenary College was host at a tea for the visitors. Friday night the joint banquet took place. Dr. Joe J. Mickle, President of Centenary College, gave a message of welcome which was responded to by Professor Z. L. Loflin for the Association, and Miss Margaret Sumrall for the Council. Professor Lester R. Ford of the Illinois Institute of Technology, gave the banquet address.

*Officers for 1949–50*

J. W. McClimans, Southeastern Louisiana College—Chairman
Miss Margaret Sumrall, Sunflower Junior College—Vice-Chairman
Miss Annie Lester, Central High School, Jackson—Secretary
Houston T. Karnes, Louisiana State University—Recorder
Miss Jessie May Hoag, Southwestern Louisiana Institute—Louisiana Representative
Miss Virginia Felder, Mississippi Southern College—Mississippi Representative

The following officers were elected for 1950–51.

Miss Margaret Sumrall, Sunflower Junior College—Chairman
L. M. Garrison, Louisiana Polytechnic Institute—Vice Chairman
Miss Elizabeth Ziegel, Delta State Teachers College—Secretary

The other officers remain the same.

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The sixth meeting of the 1949–50 season of the Men's Mathematics Club of Chicago, Ill., was held on Friday, March 17, 1950. Dr. L. R. Wilcox, Professor of Mathematics, Illinois Institute of Technology, spoke on "The Postulational Method in Mathematics."

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Miss Sarah Ruby, of Jefferson High School, Portland, Oregon, is retiring after forty-one years of teaching mathematics. For some years Miss Ruby has worked untiringly as Oregon state representative of The National Council of Teachers of Mathematics. THE MATHEMATICS TEACHER takes this opportunity to thank her for the fine job well done and to wish her well in her retirement.

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## MATHEMATICS DIVISION OF THE ALABAMA EDUCATION ASSOCIATION

President—Miss Elizabeth Eich, Decatur, Ala.
Vice President—Miss Jeanette Garrett, Birmingham, Ala.
Secretary-Treasurer—J. C. Vaughn, Mobile, Ala.

Thursday, March 30, 1950. 9:30 A.M. Small auditorium, Phillips High School, Birmingham, Ala.

9:30—Business; election of officers
10:00—National Council of Teachers of Mathematics, J. E. Allen, Birmingham, Ala.
10:10—Electronic Computers Related to Basic High-School and College Mathematics, A demonstration lecture displaying specific computers developed at R.C.A. Laboratories, Dr. Cyril N. Hayler, Radio Corporation of America, Laboratories Division, Princeton, N. J.

*Officers for 1950–51*

President—Miss Jeanette Garrett, Phillips High School, Birmingham, Ala.
Vice President—Mrs. E. L. Ashbury, Holt High School, Holt, Ala.
Secretary-Treasurer—Miss Lila Pritchett, Decatur High School, Decatur, Ala.

# Program
# The National Council of Teachers of Mathematics

Tenth Summer Meeting
The University of Wisconsin
Madison, Wisconsin
August 20, 21, 22, 23, 24, 1950

Theme: The Mathematics Teacher at Work in the Classroom, in the School, and through Organizations.

Special credit for assistance in planning this program is due the Illinois Council of Teachers of Mathematics, the Indiana Council of Teachers of Mathematics, the Iowa Association of Mathematics Teachers, the Minnesota Council of Teachers of Mathematics, and the Wisconsin Mathematics Council. Each of the first four state groups is sponsoring a section in the program. The Wisconsin Mathematics Council is host organization in co-operation with the Department of Mathematics and the School of Education of The University of Wisconsin.

## Sunday, August 20, 1950

3:00 to 8:00 P.M. Registration. Barnard Hall, University Avenue and Park Street.

The sessions of the Tenth Summer Meeting will be held in The Wisconsin Union on Lake Mendota at Park and Langdon Streets.

## Monday, August 21, 1950

8:00 A.M. to 6:00 P.M. Registration. The Wisconsin Union.

9:00 to 10:00 A.M. General Address followed by Discussion (Elementary School).

*Arithmetic—Tool Subject or Content Subject.* W. A. Brownell, Northwestern University, Evanston, Illinois

9:00 to 10:00 A.M. Junior High School Section.

*Junior High School Mathematics Adjusted to Adolescent Characteristics.* Philip Peak, University High School, Indiana University, Bloomington, Indiana.

*Problem Solving Can Be Fun.* Alice M. Hach, University High School, University of Michigan, Ann Arbor, Michigan.

9:00 to 10:00 A.M. General Address followed by Discussion (Secondary School).

*How to Solve Problems—A Requirement in Mathematics and a Requirement in Life.* E. H. C. Hildebrandt, Northwestern University, Evanston, Illinois.

9:00 to 10:00 A.M. Teacher Education Section.

*Toward Better Teaching of Mathematics.* Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan.

*Arithmetic and the Mature Mind.* Elinor B. Flagg, Illinois State Normal University, Normal, Illinois.

10:30 A.M. to 12:00 M. Curriculum Section. This section is sponsored by the *Iowa Association of Mathematics Teachers.* *Adjusting the Junior High School Curriculum in Light of Recent Trends in the Elementary School.* Panel Chairman: Irvin H. Brune, Iowa State Teachers College, Cedar Falls, Iowa. Members: Ralph Aschenbrenner, Iowa City High School, Iowa City, Iowa; Frieda Blum, Junior High School, Fort Dodge, Iowa; Lucille Houston, Oak Street Junior High School, Burlington, Iowa; Vernon Kirlin, Principal, Grant and Wilson Elementary

Schools, Mason City, Iowa; Katherine Walker, Roosevelt Junior High School, Mason City, Iowa.

10:30 to 11:50 A.M. Elementary School Section

*The Individual Interview as a Technique for Discovering Pupils' Difficulties in Arithmetic.* D. Banks Wilburn, Dean of Teachers College, Marshall College, Huntington, West Virginia.

*Teaching Aids in Elementary Arithmetic.* Alice Rose Carr, Ball State Teachers College, Muncie, Indiana.

10:30 to 11:50 A.M. Geometry Section.

*Tenth Year Mathematics as New York State Sees It.* George Grossman, DeWitt Clinton High School, New York, New York.

*Only Originals Justify Geometry.* Ona Kraft, Collinwood High School, Cleveland, Ohio.

10:30 to 11:50 A.M. College Section.

*Junior College Mathematics with a Purpose.* John G. Bowker, Middlebury College, Middlebury, Vermont.

*Objectives in Preparing Examination Questions.* E. Richard Heineman, Texas Technological College, Lubbock, Texas.

1:30 to 2:30 P.M. Slow High School Freshman Section. This program is sponsored by the *Illinois Council of Teachers of Mathematics.* Chairman: Henry Swain, New Trier Township High School, Winnetka, Illinois.

*How Can We Make General Mathematics Interesting to the Slow High School Freshman?* Victoria Larson, Waukegan Township High School, Waukegan, Illinois.

1:30 to 2:30 P.M. Elementary School Section.

*Using What We Know About Children in Developing a Mathematics Program for the Elementary School.* Josephine Kremer, Elementary Supervisor, Austin, Minnesota.

*Meaning and Methods in Arithmetic.* Dale Carpenter, Mathematics Supervisor, Los Angeles, California.

1:30 to 2:30 P.M. Algebra Section.

*Greater Pupil Participation in the College Preparatory Algebra Classes.* Dorothy S. Wheeler, Bulkeley High School, Hartford, Connecticut.

*How to Rationalize Algebraic Concepts and Processes.* Elizabeth H. Zeigel, Delta State Teachers College, Cleveland, Mississippi.

1:30 to 2:30 P.M. General Address.

*The Methods of Archimedes on Areas and Volumes.* R. E. Langer, The University of Wisconsin, Madison, Wisconsin.

2:45 to 4:15 P.M. Laboratory Section.

*Special Activities to Add to Understanding and Interest in Mathematics.* Sophia McDonald, University of California, Berkeley, California.

*The Mathematics Classroom as a Laboratory.* Donovan Johnson, University of Minnesota, Minneapolis, Minnesota.

2:45 to 4:00 P.M. Discussion Groups.

Group A1. Topic: *Mathematics for the Many—Current Approaches and Problems.*

Leader: William Betz, Mathematics Specialist, Rochester, New York.

Group A2. Topic: *Field Work as an Extracurricular Activity in Plane Geometry and Trigonometry.*

Leader: Sadie Clapper, Springfield High School, Springfield, Illinois.

Group A3. Topic: *Ways to Enrich the Mathematics Program in the Junior High School.*

Leader: Elenore Lazansky, Claremont Junior High School, Oakland, California.

Group A4. Topic: *Teaching Individuals in the Mathematics Class.*

Leader: Merle M. Ohlsen, College of Washington, Pullman, Washington.

Group A5. Topic: *The Blight of Mathematical Slang.*

Leader: Lurline Stewart, Hinds Junior College, Raymond, Mississippi.

Group A6. Topic: *Evaluating the Learning of Meanings in Arithmetic.*
Leader: W. B. Storm, Northern Illinois State Teachers College, De Kalb, Illinois.

Group A7. Topic: *What Arithmetic Teaching Techniques Succeed.*
Leader: Joseph J. Urbancek, Chicago Teachers College, Chicago, Illinois.

2:45 to 4:00 P.M. Get-Together for wives and children of those attending the Council meeting.

4:00 to 5:00 P.M. Showing of Mathematics Films.

6:00 P.M. Picnic Supper at University Picnic Point on Lake Mendota.

7:15 to 8:30 P.M. Boat Excursion on Lake Mendota.

## Tuesday, August 22, 1950

8:00 A.M. to 6:00 P.M. Registration.

9:00 to 10:00 A.M. Junior High School Section. This program is sponsored by the *Illinois Council of Teachers of Mathematics.* Chairman: Mildred Nielsen, Cossitt School, LaGrange, Illinois.

*What Are Some of the Teaching Aids that We Can Use to Make Mathematics More Meaningful to Junior High School Students*? Fay Layne, The University of Chicago, Chicago, Illinois.

9:00 to 10:00 A.M. General Address followed by Discussion (Elementary School).

*Critical Problems Concerning Arithmetic Teaching.* Ben A. Sueltz, State Teachers College, Cortland, New York.

9:00 to 10:00 A.M. General Address followed by Discussion (Senior High School).

*Resources Available to the Mathematics Teacher.* Veryl Schult, Director of Mathematics, Washington, D. C.

9:00 to 10:00 A.M. General Address followed by Discussion.

*Realism in the Training of Mathematics Teachers.* Maurice L. Hartung, The University of Chicago, Chicago, Illinois.

10:30 to 11:50 A.M. Elementary Section. This program is sponsored by the *Illinois Council of Teachers of Mathematics.* Chairman: W. B. Storm, Northern Illinois State Teachers College, De Kalb, Illinois.

*What Are Some of the Teaching Aids that We Can Use to Make Arithmetic More Meaningful*? Mildred B. Cole, C. M. Bardwell School, Aurora, Illinois.

10:30 A.M. to 12:00 M. Curriculum Section. This program is sponsored by the *Minnesota Council of Teachers of Mathematics.*

*Plans for a New Curriculum in Mathematics in Minnesota.* Panel Chairman: Edith Woolsey, Sanford Junior High School, Minneapolis. Members: Donovan Johnson, University of Minnesota, Minneapolis, Minnesota; Harry C. Johnson, University of Minnesota, Duluth, Minnesota; Clarice Kaasa, High School, Red Wing, Minnesota; Louise Kinn, Franklin Junior High School, Brainerd, Minnesota; Ethel Saupe, High School, Tracy, Minnesota; Kenneth Wegner, Carleton College, Northfield, Minnesota.

10:30 to 11:50 A.M. Senior High School Section.

*The Heart of Algebra.* Kenneth Brown, University of Tennessee, Knoxville, Tennessee.

*Senior Mathematics for the College Bound Student.* Madeline Messner, Abraham Clark High School, Roselle, New Jersey.

10:30 to 11:50 A.M. College Section.

*Critical Thinking as an Aim in Mathematics Courses for General Education.* J. M. Zant, Oklahoma A & M College, Stillwater, Oklahoma.

*General Education Mathematics on the College Level.* F. N. Fisch, Colorado

State College of Education, Greeley, Colorado.

1:30 to 2:30 P.M. General Address followed by Discussion (Elementary School).

*Better Practices in Arithmetic Teaching.* Harry C. Johnson, University of Minnesota, Duluth Branch, Duluth, Minnesota.

1:30 to 2:30 P.M. In-Service Training Section.

*The 1950 in-Service Mathematics Workshop in Wilmington, Delaware.* Florence E. Loose, Wilmington High School, Wilmington, Delaware.

*In-Service Growth.* Agnes Herbert, Clifton Park Junior High School, Baltimore, Maryland.

1:30 to 2:30 P.M. General Address.

*Linear Equations Without Determinants, Part I.* C. C. MacDuffee, The University of Wisconsin, Madison, Wisconsin.

1:30 to 2:30 P.M. Affiliated Groups Section.

*How Can Affiliated Groups Cooperate for Improvement of Mathematics Teaching?*

Leader: Harry W. Charlesworth, East High School, Denver, Colorado

Discussants: Presidents of State Councils.

Illinois: Helen A. Schneider, Oak School, LaGrange, Illinois.

Indiana: K. Eileen Beckett, Lebanon High School, Lebanon, Indiana.

Iowa: Glenadine Gibb, Iowa State Teachers College, Cedar Falls, Iowa.

Kansas: Charles B. Tice, High School, Abilene, Kansas.

Minnesota: Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota.

Nebraska: Maude Holden, High School, Ord, Nebraska.

Wisconsin: John Mayor, The University of Wisconsin, Madison, Wisconsin.

2:45 to 4:15 P.M. Secondary School Section.

*How to Use the Mathematics Classroom for Intercultural Education.* Paul Cantonwine, The Lakeside School, Seattle, Washington.

*Ten Great American Men of Mathematics.* Edwin W. Schreiber, Western Illinois State College, Macomb, Illinois.

2:45 to 4:00 P.M. Discussion Groups.

Group B1. Topic: *How Can the College Best Prepare Students for Teaching Mathematics in the High School?*

Leader: Henriette L. Brudos, State Teachers College, Valley City, North Dakota.

Group B2. Topic: *Teaching Theorems as Originals Increases Interest in Geometry.*

Leader: Gladys R. Clark, Normandy High School, Normandy, Missouri.

Group B3. Topic: *Have We Raised or Lowered Our Achievement in the Elementary School in the Past Ten Years?*

Leader: Mary Entsminger, University School, Southern Illinois University, Carbondale, Illinois.

Group B4. Topic: *An Educational Approach to Desirable Mathematical Attitudes Among Non-Mathematicians.*

Leader: Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana.

Group B5. Topic: *Mathematics for the Neglected Group of High School Pupils.*

Leader: Andrew Nelsen, Central High School, Omaha, Nebraska.

Group B6. Topic: *Solutions to Verbal Problems in the Junior High School—Tricks of Magic or Products of Reasoning.*

Leader: Frances M. Smith, Thomas Jefferson High School, Passaic, New Jersey.

Group B7. Topic: *How to Enrich the Teaching of Secondary Mathematics by the Use of Multi-Sensory Aids.*

Leader: Izetta Sparks, North Texas State College Demonstration School Denton, Texas.

4:00 to 5:00 P.M. Showing of Mathematics Films.

4:15 to 5:30 P.M. Tour of Madison.

6:30 to 8:30 P.M. Terrace Supper, The Wisconsin Union Terrace. Host Organization: Milwaukee County Suburban Teachers Club.

## Wednesday, August 23, 1950

8:00 A.M. to 6:00 P.M. Registration.

9:00 to 10:00 A.M. General Address followed by Discussion (Elementary School).

*What Makes Arithmetic Meaningful?* H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa.

9:00 to 10:00 A.M. General Address and Discussion (Junior High School).

*Making Junior High School Mathematics Interesting.* Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota. (Those attending will be given an opportunity to tell about one plan or device which has added to the interest in some class.)

9:00 to 10:00 A.M. History of Mathematics Section.

*New Emphases in the History of Mathematics.* Vera Sanford, State Teachers College, Oneonta, New York.

9:00 to 10:00 A.M. Junior College Section.

*What Courses Should We Teach?* J. Stuart McNair, Canal Zone Junior College, Balboa, Canal Zone.

*Content and Objectives of Junior College Mathematics.* H. Glenn Ayre, Western Illinois State College, Macomb, Illinois.

10:15 A.M. to 12:15 P.M. Geometry Section.

This program is sponsored by the *Indiana Council of Teachers of Mathematics.*

*The Changes in Geometry.* Panel Chairman: K. Eileen Beckett, Lebanon, Indiana, President.

*Enrichment Materials,* K. Eileen Beckett.

*Sense Aids,* Walter H. Carnahan, Purdue University, Lafayette, Indiana.

*Recent Trends in Geometry,* Wilma Flewelling, Horace Mann High School, Gary, Indiana.

*Exploration in Geometry,* Paul L. Trump, The University of Wisconsin, Madison, Wisconsin.

10:30 to 11:50 A.M. Junior High School Section.

*How General Should General Mathematics Be?* Marie S. Wilcox, George Washington High School, Indianapolis, Indiana.

*New York's New Program in Junior High School Mathematics.* Ben A. Sueltz, State Teachers College, Cortland, New York.

10:30 to 11:50 A.M. Senior High School Section.

*Digressions in Arithmetic.* Harold D. Larsen, Albion College, Albion, Michigan.

*The Enrichment of High School Mathmatics.* L. E. Bush, College of St. Thomas, St. Paul, Minnesota.

10:30 to 11:50 A.M. College Section.

*Another Course for College Freshmen.* H. Vernon Price, State University of Iowa, Iowa City, Iowa.

*A Course in Mathematics for Non-Mathematics Majors in Teacher Training Institutions.* Aaron Bakst, School of Education, New York University New York, New York.

1:30 to 2:30 P.M. Elementary School Section.

*The Operational Theory of Meaning and Its Importance for Use of an Abacus in the Teaching of Arithmetic.* Nathan Lazar, The Ohio State University, Columbus, Ohio.

1:30 to 2:30 P.M. General Section.

*Linear Equations without Determinants,*

*Part II.* C. C. MacDuffee, The University of Wisconsin, Madison, Wisconsin.

1:30 to 2:30 P.M. Exhibits and Contests Section.

*Exhibits, Contests, and Interest Stimulation.* Walter H. Carnahan, Purdue University, Lafayette, Indiana.

2:45 to 4:00 P.M. Solid Geometry Section.

*How to Get the Most Out of Solid Geometry.* Rachel Keniston, Stockton College, Stockton, California.

2:45 to 4:00 P.M. Discussion Groups.

Group C1. Topic: *Consumer Mathematics —A Study in Effective Citizenship.*
Leader: R. A. Baumgartner, Freeport High School, Freeport, Illinois.

Group C2. Topic: *Teaching Thought Problems in First Year Algebra.*
Leader: Elbert Fulkerson, University High School, Southern Illinois University, Carbondale, Illinois.

Group C3. Topic: *Tenth Year General Mathematics.*
Leader: George Grossman, DeWitt Clinton High School, New York, New York.

Group C4. Topic: *A Better Vocabulary for Teaching Arithmetic.*
Leader: Wilbur Harris, Glenwood School, Findlay, Ohio.

Group C5. Topic: *What Can We Do for the Exceptional Pupils?*
Leader: George Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois.

Group C6. Topic: *How About Teaching for Transfer of Arithmetic to the Unknown Problems of the Future.*
Leader: Gertrude Hendrix, Eastern Illinois State College, Charleston, Illinois.

Group C7. Topic: *Methods of Presenting Geometric Concepts by Means of Visual Aids.*
Leader: Maude Holden, High School, Ord, Nebraska.

4:00 to 5:00 P.M. Showing of Mathematics Films.

6:30 P.M. Banquet. Tripp Commons.
Host organization: To be announced later

*From the Pupil's Seat.* Mary A. Potter, Mathematics Supervisor, Racine, Wisconsin

*50 Years of Mathematical Reform—An Attempt at Evaluation.* William Betz, Mathematics Specialist, Rochester, New York

## Thursday, August 24, 1950

9:30 A.M. Buses will leave the Wisconsin Union.

11:00 A.M. Picnic Lunch at Devil's Lake State Park near Baraboo, Wisconsin.

12:30 P.M. Boat Trip to Upper Dells of the Wisconsin River.

4:00 P.M. Buses return to Madison.

If a sufficient number wish to remain for the Indian Ceremonial at 8:00 P.M., transportation can be arranged.

### ANNOUNCEMENTS

*Trip to Devil's Lake State Park and the Dells of the Wisconsin River.* Stops will be made at points of interest including Devil's Lake State Park, famous for its geological oddities caused by glacial action. The 15 mile river trip through the Upper Dells, includes shore landings at Stand Rock, Witches Gulch and Cold Water Canyon. The boat trip will start about 12:30 P.M. The Dells, sometimes called the "Scenic Wonderland of the Middlewest," is also of much interest geologically.

The cost of $4.00 includes transportation in University buses, box luncheon prepared by the University Residence Halls, and the boat trip through the Upper Dells.

*Terrace Supper and Banquet Reservations.* Reservations for the Terrace Supper on Tuesday, August 22, and the Banquet on Wednesday, August 23, should be made in advance by sending check or money order with the Registration and Reservation Form. The price for the

Terrace Supper is $1.50 and for the banquet is $2.00. Reservation requests received by August 10 will be acknowledged by mail. All requests for Terrace Supper and Banquet Reservations should be sent to J. R. Mayor, North Hall, The University of Wisconsin, Madison 6, Wisconsin.

Reservations for the Picnic on Monday, August 21, for those who are not staying in Barnard or Chadbourne Halls, should also be made in advance. The price of the Picnic Supper is $1.15.

*Accommodations in University Residence Halls.* The University of Wisconsin's Division of Residence Halls has made available two campus residences, Barnard and Chadbourne Halls, as the living quarters for those attending the Council meetings. Reservations may be made for the period Sunday, August 20, through Thursday, August 24. Meal service will begin with breakfast Monday, August 21, and end with luncheon Wednesday, August 23. (The evening meals on Tuesday and Wednesday will be served elsewhere.) The University will provide maid service, linens, towels and soap. Most of the rooms are doubles and must be used as such; if you express no choice of roommate you will be paired with someone chosen at random.

With services provided as outlined above, the charge for those staying through Wednesday night, August 23, will be $13.25 per person. This total includes the Picnic on Monday night, which is served by the Residence Halls staff, but does not include the evening meals on Tuesday and Wednesday. Those desiring lodging Thursday night, August 24 will be accommodated for an additional $2.00 per person. Children six years of age or under will be accomodated for one-half of these rates.

Guests arriving late or who indicate upon arrival that they must leave early will pay only for the services they are to receive.

Reservations for Residence Halls accommodations may be made by mailing the Reservation and Registration Form not later than August 1. Room and board bills will be collected by members of the Division of Residence Halls staff as guests arrive.

*Hotels.* Inquiries about and requests for reservations in hotels may be addressed directly to Mr. Otto Mueller, Director of Housing, The University of Wisconsin, 434 Sterling Court, Madison 6, Wisconsin. Mr. Mueller will arrange hotel reservations for those who wish to take advantage of his office for this service.

*Meal Service at The Wisconsin Union.* Meals may be obtained at The Wisconsin Union by those who are not staying in Barnard and Chadbourne Halls. The Union will be open for the period of The National Council Meeting in order to provide meeting rooms for the various sessions and to serve the Terrace Supper and Banquet.

*Discussion Groups.* Requests for attendance at these meetings should be made in advance. First and second choices may be indicated on the attached Reservation and Registration Form. Admittance cards will be returned by mail to those making their requests before August 10, 1950. Admittance to these groups will be by admission card only.

*Registration.* There will be a registration fee of fifty cents for members of the National Council, members of the Mathematical Association of America, and for teachers in the elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students, sponsored by a faculty member; relatives of members; invited speakers who are not members; members of the press; and exhibitors are admitted without payment of registration fee, but should register at the registration desk.

*Exhibits.* There will be an exhibit of mathematical models, instruments, teaching aids, and other classroom materials. Teachers are invited to bring materials for exhibit. If you are willing to bring materials from your school please com-

municate in advance with Lawrence Wahlstrom, 1320 Rust Street, Eau Claire, Wisconsin.

Many free and inexpensive teaching materials will also be on exhibit. Suggestions of recent materials which have become available and have classroom value should be sent to Lola Pierstorff, School of Education Library, The University of Wisconsin, Madison 6, Wisconsin.

*Commercial Exhibits.* Textbooks and Commercial Teaching Aids will be on exhibit. Inquiries for exhibit space should be addressed to Professor Ralph Huffer, Beloit College, Beloit, Wisconsin.

*Supplies and Equipment.* Speakers and other participants in the program who need blackboards, projection equipment or other materials should communicate not later than August 10 with John A. Brown, Wisconsin High School, Madison 6, Wisconsin.

*Films and Filmstrips.* Persons wishing to view specific films while in attendance at the meeting should communicate with Kenneth Fish, 1756 Congress Avenue, Beloit, Wisconsin, not later than August 1.

*Mail and Telegrams.* Mail and telegrams for those attending the meetings should be addressed in care of The National Council of Teachers of Mathematics, North Hall, Madison 6, Wisconsin or directly to the person at his address for the meeting if it is known.

For points not covered in this announcement please write directly to J. R. Mayor, North Hall, Madison 6, Wisconsin.

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RESERVATION AND REGISTRATION FORM

Please fill out both sides of the form below and mail to J. R. Mayor, North Hall, Madison 6, Wisconsin.

Name _____
Last Name First Name Middle Initial

Home Address _____
Street and Number City Zone State

School Address _____
Name of School City State

Indicate with an X in front of address, the one to which tickets should be sent. Member of N.C.T.M. Yes ( ) No ( ); M.A.A. Yes ( ) No ( ); Student ( )

Please check your fields of interest:

( ) Elem. ( ) Jr. H.S. ( ) High School ( ) Jr. Col. ( ) College
( ) Teacher Training ( ) Supervision

Members of families and friends of those attending the meetings are welcome at all social and recreational events.

Registration Fee $0.50 or $1.50 Number_____ Amount Enclosed_____

Picnic Monday $1.15 No. of Tickets_____ Amount Enclosed
(If you plan to stay at Barnard or Chadbourne Hall do *not* send money for Picnic in advance)

Terrace Supper Tuesday $1.50 No. of Tickets _____ Amount Enclosed _____

Banquet Wednesday $2.00 No. of Tickets _____ Amount Enclosed _____

Trip to Dells Thursday $4.00 No. of Tickets _____ Amount Enclosed _____

I enclose (check, money order) for $ _____ to pay for the above.
(Checks or money orders should be made payable to Arnold Wendt, Treasurer, Madison meeting.)

(*over*)

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DISCUSSION SECTIONS:

My choice for discussion sessions is as follows: (Please insert name of leader and number of group as listed on program.)

Monday: 1st choice: Group _____ Leader _____

2nd choice: Group _____ Leader _____

Tuesday: 1st choice: Group _____ Leader _____

2nd choice: Group _____ Leader _____

Wednesday: 1st choice: Group _____ Leader _____

2nd choice: Group _____ Leader _____

ACCOMMODATIONS IN RESIDENCE HALLS

Please reserve accommodations in Residence Halls as follows:

_____ Myself only. I wish to room with _____

_____ Myself and wife or husband.

_____ Myself and following named members of my family:

_____, _____, _____, _____

(Give ages of children)

Will arrive in Madison on _____, depart on _____

No payment before arrival in Madison is necessary for accommodations in University Residence Halls.